

1 **SUPPLEMENTARY INFORMATION TO:**

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4 **New particle formation from sulfuric acid and ammonia: nucleation and**  
5 **growth model based on thermodynamics derived from CLOUD**  
6 **measurements for a wide range of conditions**

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## 13 **SI Text1: Acid-base model differential equations**

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15 The model integrates the differential equations listed below using a fourth-order Runge-  
16 Kutta algorithm. The time-dependent concentration for each cluster depends on the balance  
17 between production rate,  $P$ , and loss rate,  $L$ . Relevant loss terms in the present study (analyzing  
18 new particle formation in the CLOUD chamber) are the wall loss rate,  $k_w$ , the dilution rate,  $k_{dil}$ ,  
19 and the condensation/coagulation sink (see Kürten et al., 2018). For the smallest clusters the  
20 evaporation rate,  $E$ , is also a relevant loss process. The clusters in the molecular size bins are  
21 denoted by their sulfuric acid content,  $A$ , where an index indicates the number of molecules  
22 contained in the cluster. For the smallest clusters (monomer up to the tetramer) the clusters are  
23 also distinguished according to their base content, where  $B$  with an index indicates the number  
24 of base (ammonia) molecules in the cluster (note that the clusters never contain more base than  
25 acid, see main text). This nomenclature results in the following set of equations.

26

### 27 **Monomers ( $A_1$ and $A_1B_1$ ):**

28

$$29 \quad \frac{dA_1B_1}{dt} = P - L \cdot A_1B_1 \quad (1a)$$

30

$$31 \quad P = K_{1,1} \cdot A_1 \cdot B_1 + E_{A_2B_1} \cdot A_2B_1 \quad (1b)$$

32

$$33 \quad L = k_{w,1} + k_{dil} + E_{A_1B_1} + \sum_{i=1}^n K_{1,i} \cdot N_i \quad (1c)$$

34

35 The last term in the equation of the losses ( $L$ ) represents the condensation/coagulation sink.

36

37 The parameter  $A_{1,total}$  is a constant input parameter determining the total sulfuric acid  
38 concentration. From this the concentration of “free” sulfuric acid is calculated in every time  
39 step of the integration:

40

$$41 \quad A_1 = A_{1,total} - A_1B_1 \quad (1d)$$

42

43

### 44 **Dimers ( $A_2$ , $A_2B_1$ and $A_2B_2$ ):**

45

$$46 \quad \frac{dA_2}{dt} = P - L \cdot A_2 \quad (2a)$$

47

$$48 \quad P = 0.5 \cdot K_{1,1} \cdot A_1 \cdot A_1 + E_{A_3B_0} \cdot A_3 \quad (2b)$$

49

$$50 \quad L = k_{w,2} + k_{dil} + E_{A_2} + K_{1,2} \cdot B_1 + \sum_{i=1}^n K_{2,i} \cdot N_i \quad (2c)$$

51

$$52 \quad \frac{dA_2B_1}{dt} = P - L \cdot A_2B_1 \quad (2d)$$

53

$$54 \quad P = K_{1,1} \cdot A_1 \cdot A_1B_1 + K_{1,2} \cdot B_1 \cdot A_2 + E_{A_2B_2} \cdot A_2B_2 + E_{A_3B_1} \cdot A_3B_1 \quad (2e)$$

55

56  $L = k_{w,2} + k_{dil} + E_{A_2B_1} + K_{1,2} \cdot B_1 + \sum_{i=1}^n K_{2,i} \cdot N_i$  (2f)

57

58

59  $\frac{dA_2B_2}{dt} = P - L \cdot A_2B_2$  (2g)

60

61  $P = 0.5 \cdot K_{1,1} \cdot A_1B_1 \cdot A_1B_1 + K_{1,2} \cdot B_1 \cdot A_2B_1 + E_{A_3B_2} \cdot A_3B_2$  (2h)

62

63  $L = k_{w,2} + k_{dil} + E_{A_2B_2} + \sum_{i=1}^n K_{2,i} \cdot N_i$  (2i)

64

65

66 **Trimers (A<sub>3</sub>, A<sub>3</sub>B<sub>1</sub>, A<sub>3</sub>B<sub>2</sub> and A<sub>3</sub>B<sub>3</sub>):**

67

68  $\frac{dA_3}{dt} = P - L \cdot A_3$  (3a)

69

70  $P = K_{1,2} \cdot A_1 \cdot A_2 + E_{A_4} \cdot A_4$  (3b)

71

72  $L = k_{w,3} + k_{dil} + E_{A_3} + K_{1,3} \cdot B_1 + \sum_{i=1}^n K_{3,i} \cdot N_i$  (3c)

73

74

75  $\frac{dA_3B_1}{dt} = P - L \cdot A_3B_1$  (3d)

76

77  $P = K_{1,2} \cdot A_1 \cdot A_2B_1 + K_{1,3} \cdot B_1 \cdot A_3 + K_{1,2} \cdot A_1B_1 \cdot A_2 + E_{A_4B_1} \cdot A_4B_1$  (3e)

78

79  $L = k_{w,3} + k_{dil} + E_{A_3B_1} + K_{1,3} \cdot B_1 + \sum_{i=1}^n K_{3,i} \cdot N_i$  (3f)

80

81

82  $\frac{dA_3B_2}{dt} = P - L \cdot A_3B_2$  (3g)

83

84  $P = K_{1,3} \cdot B_1 \cdot A_3B_1 + K_{1,2} \cdot A_1 \cdot A_2B_2 + K_{1,2} \cdot A_1B_1 \cdot A_2B_1 + E_{A_4B_2} \cdot A_4B_2 + E_{A_3B_3} \cdot A_3B_3$   
 85 (3h)

86

87  $L = k_{w,3} + k_{dil} + E_{A_3B_2} + K_{1,3} \cdot B_1 + \sum_{i=1}^n K_{3,i} \cdot N_i$  (3i)

88

89

90  $\frac{dA_3B_3}{dt} = P - L \cdot A_3B_3$  (3j)

91

92  $P = K_{1,3} \cdot B_1 \cdot A_3B_2 + K_{1,2} \cdot A_1B_1 \cdot A_2B_2 + E_{A_4B_3} \cdot A_4B_3$  (3k)

93

94  $L = k_{w,3} + k_{dil} + E_{A_3B_3} + \sum_{i=1}^n K_{3,i} \cdot N_i$  (3l)

95

96

97 **Tetramers (A<sub>4</sub>, A<sub>4</sub>B<sub>1</sub>, A<sub>4</sub>B<sub>2</sub>, A<sub>4</sub>B<sub>3</sub> and A<sub>4</sub>B<sub>4</sub>):**

98

99 
$$\frac{dA_4}{dt} = P - L \cdot A_4 \quad (4a)$$

100

101 
$$P = K_{1,3} \cdot A_1 \cdot A_3 + 0.5 \cdot K_{2,2} \cdot A_2 \cdot A_2 \quad (4b)$$

102

103 
$$L = k_{w,4} + k_{dil} + E_{A_4} + K_{1,4} \cdot B_1 + \sum_{i=1}^n K_{4,i} \cdot N_i \quad (4c)$$

104

105

106 
$$\frac{dA_4B_1}{dt} = P - L \cdot A_4B_1 \quad (4d)$$

107

108 
$$P = K_{1,4} \cdot B_1 \cdot A_4 + K_{1,3} \cdot A_1 \cdot A_3B_1 + K_{1,3} \cdot A_1B_1 \cdot A_3 + K_{2,2} \cdot A_2 \cdot A_2B_1 \quad (4e)$$

109

110 
$$L = k_{w,4} + k_{dil} + E_{A_4B_1} + K_{1,4} \cdot B_1 + \sum_{i=1}^n K_{4,i} \cdot N_i \quad (4f)$$

111

112

113 
$$\frac{dA_4B_2}{dt} = P - L \cdot A_4B_2 \quad (4g)$$

114

115 
$$P = K_{1,4} \cdot B_1 \cdot A_4B_1 + K_{1,3} \cdot A_1 \cdot A_3B_2 + K_{1,3} \cdot A_1B_1 \cdot A_3B_1 + K_{2,2} \cdot A_2 \cdot A_2B_2 + 0.5 \cdot K_{2,2} \cdot A_2B_1 \cdot A_2B_1 \quad (4h)$$

117

118 
$$L = k_{w,4} + k_{dil} + E_{A_4B_2} + K_{1,4} \cdot B_1 + \sum_{i=1}^n K_{4,i} \cdot N_i \quad (4i)$$

119

120

121 
$$\frac{dA_4B_3}{dt} = P - L \cdot A_4B_3 \quad (4j)$$

122

123 
$$P = K_{1,4} \cdot B_1 \cdot A_4B_2 + K_{1,3} \cdot A_1 \cdot A_3B_3 + K_{1,3} \cdot A_1B_1 \cdot A_3B_2 + K_{2,2} \cdot A_2B_1 \cdot A_2B_2 + E_{A_4B_4} \cdot A_4B_4 \quad (4k)$$

125

126 
$$L = k_{w,4} + k_{dil} + E_{A_4B_3} + K_{1,4} \cdot B_1 + \sum_{i=1}^n K_{4,i} \cdot N_i \quad CS_{A_4B_3} \quad (4l)$$

127

128

129 
$$\frac{dA_4B_4}{dt} = P - L \cdot A_4B_4 \quad (4m)$$

130

131 
$$P = K_{1,4} \cdot B_1 \cdot A_4B_3 + K_{1,3} \cdot A_1B_1 \cdot A_3B_3 + 0.5 \cdot K_{2,2} \cdot A_2B_2 \cdot A_2B_2 \quad (4n)$$

132

133 
$$L = k_{w,4} + k_{dil} + E_{A_4B_4} + \sum_{i=1}^n K_{4,i} \cdot N_i \quad (4o)$$

134

135 From the concentrations of the acid-base clusters the concentrations of the total monomer ( $N_1$ ),  
136 dimer ( $N_2$ ), trimer ( $N_3$ ) and tetramer ( $N_4$ ) can be determined:

137

138  $N_1 = A_1 + A_1 B_1$  (5a)

139  
140  $N_2 = A_2 + A_2 B_1 + A_2 B_2$  (5b)

141  
142  $N_3 = A_3 + A_3 B_1 + A_3 B_2 + A_3 B_3$  (5c)

143  
144  $N_4 = A_4 + A_4 B_1 + A_4 B_2 + A_4 B_3 + A_4 B_4$  (5d)

145  
146 The calculation of the larger clusters (starting with the pentamer) and particles is calculated in  
147 the same way as described previously (Kürten et al., 2018):

148  
149  $\frac{dN_{k \geq 5}}{dt} = P - L \cdot N_{k \geq 5}$  (5e)

150  
151  $P = \frac{1}{2} \cdot \sum_{i+j=k} N_i \cdot N_j$  (5f)

152  
153  $L = k_{w,k} + k_{dil} + \sum_{j=1}^n K_{k,j} \cdot N_j$  (5g)

154  
155

156 **SI Text2: Calculation of evaporation rates**

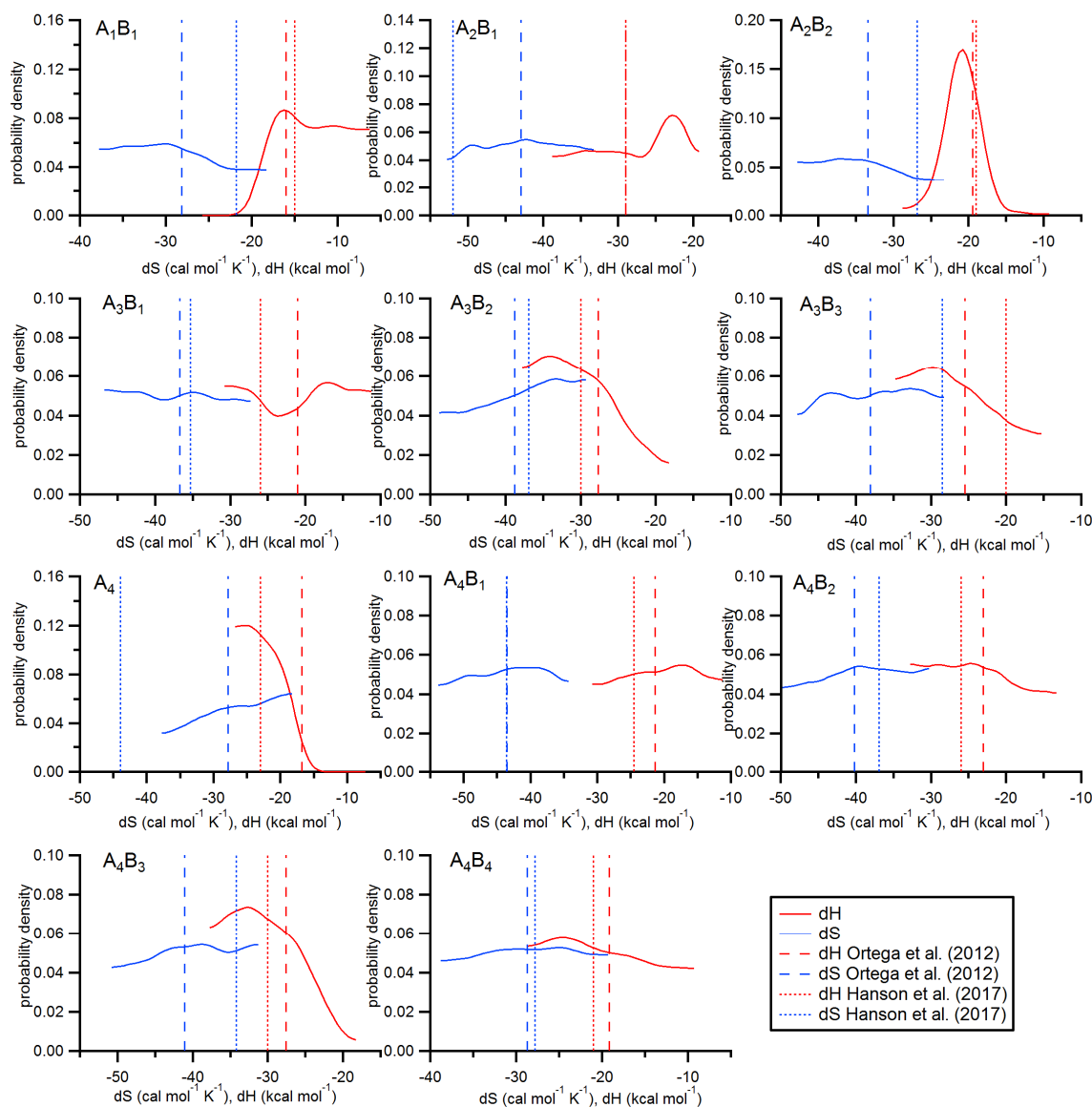
157  
158 Evaporation rates are calculated from  $dH$  (in kcal mol<sup>-1</sup>) and  $dS$  (in cal mol<sup>-1</sup> K<sup>-1</sup>) according to  
159 (Ortega et al., 2012, Kürten et al., 2015):

160  
161  $E = \frac{K \cdot 10^{-6}}{k_B \cdot T \cdot K_{eq}}$  (6)

162  
163 The equilibrium constant,  $K_{eq}$ , is given by

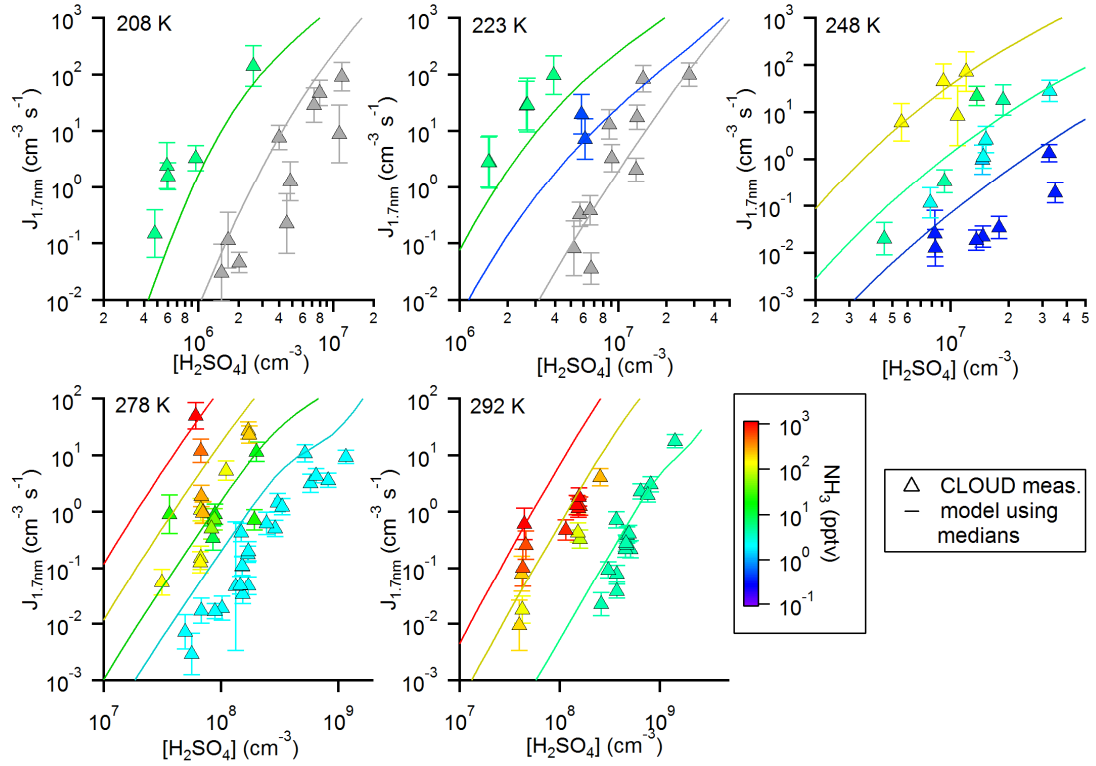
164  
165  $K_{eq} = \frac{1}{10^5 \text{ Pa}} \cdot \exp\left(-\frac{dH \cdot 1000}{R'_{gas} \cdot T} + \frac{dS}{R'_{gas}}\right)$  (7)

166  
167 The constants are  $k_B = 1.381 \times 10^{23} \text{ J K}^{-1}$  and  $R'_{gas} = 1.987 \text{ cal mol}^{-1} \text{ K}^{-1}$ ;  $T$  is the temperature and  
168  $K$  the collision rate constant (Chan and Mozurkewich, 2001); the scaling factor  $10^6$  is used to  
169 convert the collision constant from  $\text{cm}^3 \text{ s}^{-1}$  to  $\text{m}^3 \text{ s}^{-1}$ .



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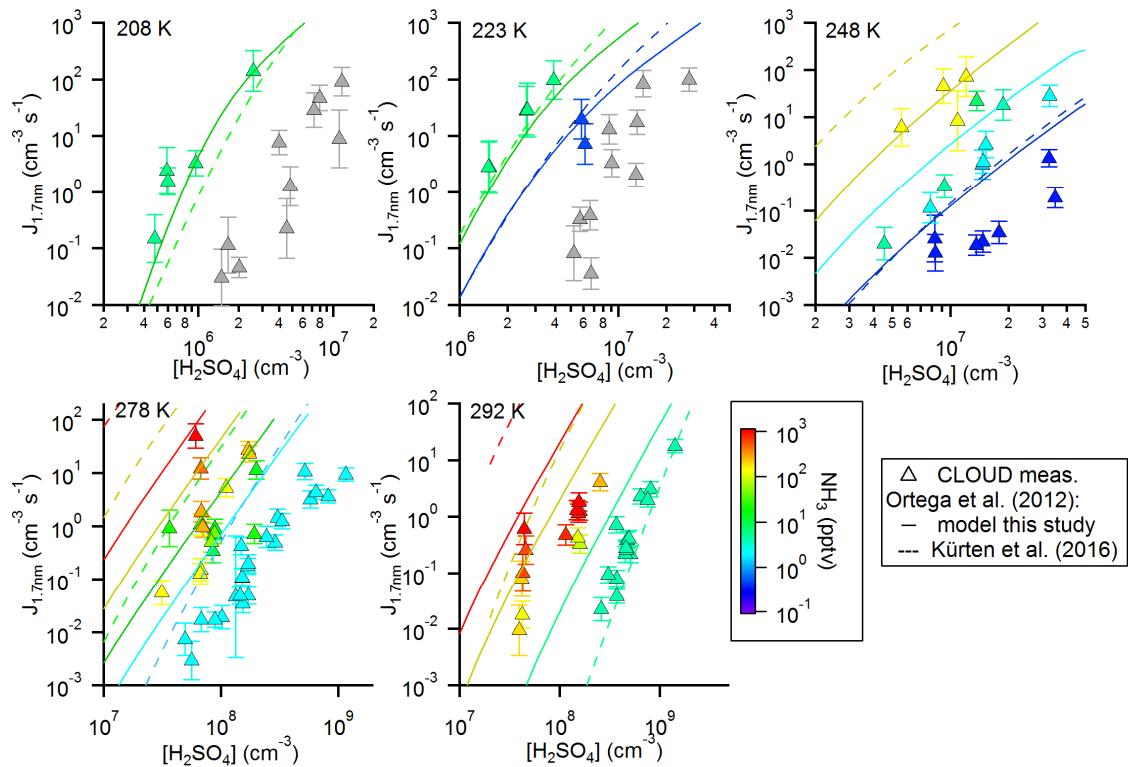
**Figure S1:** Probability density functions of  $dH$  and  $dS$  values for 11 clusters in the acid base system ( $A_xB_y$  = cluster of sulfuric acid and ammonia with  $x$  sulfuric acid molecules and  $y$  ammonia molecules). The solid curves show the results from the Monte Carlo simulation. The vertical lines indicate the literature data from Ortega et al. (2012, dashed lines) and Hanson et al. (2017, dotted lines).



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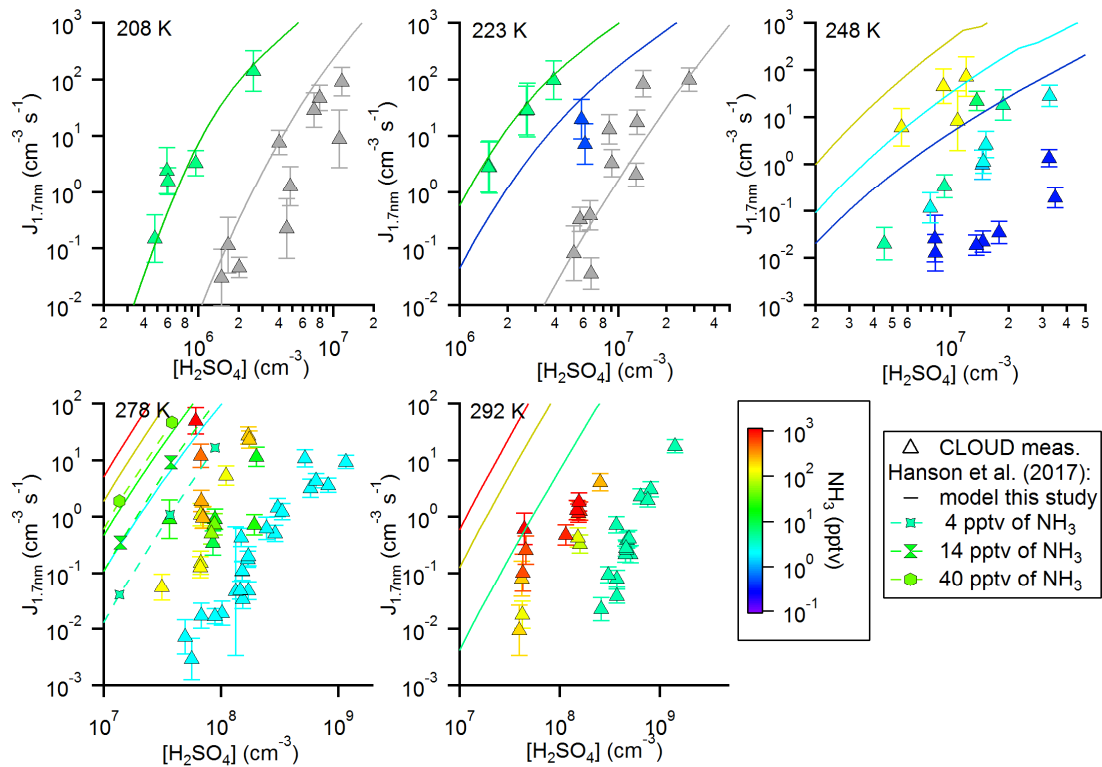
181 **Figure S2:** Comparison between simulated and measured new particle formation rates for five  
 182 different temperatures. The color code indicates the ammonia mixing ratio; the grey symbols  
 183 indicate pure binary conditions. The model uses the median values from Table 1 as the  
 184 thermodynamic data (see also Figure 2).



185  
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187 **Figure S3:** Comparison between simulated and measured new particle formation rates for five  
 188 different temperatures. The color code indicates the ammonia mixing ratio; the grey symbols  
 189 indicate pure binary conditions. The solid lines show results for the thermodynamic data from  
 190 Ortega et al. (2012) implemented in the model of the present study. The dashed lines show the  
 191 calculated NPF rates published previously by using the ACDC model with the same  
 192 thermodynamic data (Kürten et al., 2016).





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**Figure S4:** Comparison between simulated and measured new particle formation rates for five different temperatures. The color code indicates the ammonia mixing ratio; the grey symbols indicate pure binary conditions. The model uses thermodynamic data from Hanson et al. (2017). The symbols (stars, hourglass symbols and hexagons) at 278 K are original data taken from Hanson et al. (2017).

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