Interactive comment on “Parameterization of subgrid aircraft emission plumes for use in large-scale atmospheric simulations” by A. D. Naiman et al.

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Received and published: 25 February 2010

We first want to thank Dr. Schumann for his helpful and constructive comments.

The problem with this model is that diffusion can only be represented qualitatively ... A further problem with eq (3) is that it assumes that diffusion is oriented in the direction of the variable \( \xi_i \). In reality, diffusion may change the position of a piece of a plume cross-section either in positive or negative direction randomly ... As a consequence of the qualitative nature of eqs. (8, 9) also the solution given in eq. (21, 22) is correct only qualitatively ... However, at the end, this may
impact the results only to a minor degree, because both the ellipsoidal and the Gaussian plume model are crude approximations to the real concentration field in a contrail and the diffusivities are not well known anyway.

As Dr. Schumann has noted, both our ellipsoidal model and his Gaussian plume model are approximations to the real concentration field in a contrail, and the diffusivities are not certain. We would like to point out that although our equation 3 is a qualitative description of the diffusion process, the direction of change that it indicates is physical. Although diffusion may “change the position of a piece of a plume cross-section either in [a] positive or negative direction randomly,” the average effect of diffusion is always to increase the area of the plume, in this case by stretching it in the $\xi_i$ direction.

**Errors in the calculation of variances:**

We appreciate this note on the formal derivation of the variances, which we had previously derived from comments in the Schumann et al. (1995) paper. After much algebraic manipulation, we note that this derivation is exactly equivalent to our equations 10-14 if the factor 2.2 in equations 11 and 13 is changed to a factor of 2. Dr. Schumann’s form in his equations 9-11 is much more clear, however, and we will use this form instead.

*Added to manuscript in Section 3.1:*

For comparison, the SPM quantities $a$, $b$, and $\theta$ have been converted to effective plume variances by projecting the plume ellipse onto the horizontal and vertical axes and solving for the skewed variance:

\[
\sigma_v^2 = \frac{a^2}{4} \cos^2 \theta + \frac{b^2}{4} \sin^2 \theta, \tag{1}
\]

\[
\sigma_h^2 = \frac{a^2}{4} \sin^2 \theta + \frac{b^2}{4} \cos^2 \theta, \tag{2}
\]
\[ \sigma_s^2 = \left( \frac{a^2}{4} - \frac{b^2}{4} \right) \cos \theta \sin \theta. \]  

(3)

Note that \( \sigma_s^2 \) can be either positive or negative, depending on the relative magnitude of \( a \) and \( b \), but we retain the notation \( \sigma_s^2 \) to compare to Dürbeck and Gerz (1996).

It should be noted that the notation \( \sigma_s^2 \) for the off-diagonal elements is misleading.

We agree that the notation \( \sigma_s^2 \) is misleading, since the sign of this term depends on the relative direction of the vertical shear. We have chosen this notation here, however, to compare to the Dürbeck and Gerz (1996) plot using this notation, as noted in the revised part of Section 3.1, above.

Moreover, we note that a factor of 2 was missing in eq. (9) of Dürbeck and Gerz (1996) which may have effected the analysis of the cross-section area \( A \).

This correction has no effect on our analysis, since we are plotting normalized area, as did Dürbeck and Gerz (1996).

At the end, however, comparisons with measurements in the atmosphere should be used to assess the validity of the approach.

We have added a comparison and figure in Section 3.2 to the dilution data from Schumann et al. (1998).

*Added to manuscript as Section 3.2 (figure is attached as Supplement):*

**Comparison with Observations**

Schumann et al. (1998) calculated the bulk dilution ratio of exhaust plumes using data from more than 70 plume observations at various plume ages from seconds to hours. For the range of ages from less than one second to \( 10^4 \) seconds, the bulk mean data
could be approximated by curve fit,

\[ N = 7000\left(\frac{t}{t_0}\right)^{0.8}, \]

where \(N\) is the dilution ratio, \(t\) is the plume age in seconds, and \(t_0 = 1\) is an arbitrary reference scale. Individual observations differed by a factor of 3 from the mean.

Figure 1 shows a comparison of the dilution predicted by the SPM and by the Konopka (1995) analytical solution with this data fit. The SPM and Konopka (1995) estimates are shown for the case 1 and case 4 conditions as described in the previous section. Dilution for the models is calculated as the plume area normalized by the initial plume area. The data fit dilution is calculated relative to \(t = 300\) seconds, a time at which the analytic plume models become applicable, since the wake vortex system of the aircraft can be assumed to have dissipated.

The dilution predicted by the SPM falls well within the factor of 3 scatter in the observational data reported by Schumann et al. (1998). The Konopka (1995) model is within the range of scatter for the higher shear case 4, but underpredicts dilution relative to observations for the lower shear case 1.

Please also note the supplement to this comment:
http://www.atmos-chem-phys-discuss.net/9/C11125/2010/acpd-9-C11125-2010-supplement.pdf

Interactive comment on Atmos. Chem. Phys. Discuss., 9, 24755, 2009.