Interactive comment on “Rocket measurements of positive ions during polar mesosphere winter echo conditions” by A. Brattli et al.

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I have looked through the comments of referee 1 and carefully compared the work of Brattli et al. with that of Kelley and Ulwick (1988) to which the referee referred. I can only conclude that Brattli et al. have performed an analysis in a fashion that is virtually identical to that of Kelley and Ulwick (I’ll refer to this paper as KU).

Firstly, KU in equation 1 relate the sampling frequency in Hz, to the wavenumber in radians/m and rocket velocity in m/s as: \( f = V_r / (2\pi k) \). They also state just above equation 1 that the wavenumber in radians/m, \( k = (2\pi l) / l \). This means that the wavelength in m, \( l = V_r / f \). They also state just below equation 2 on that page that in their case, \( V_r = 450 \) m/s.
It should be noted that \( f \), above, and all frequencies presented in the KU paper are Hz, not angular frequency, \( \omega \), in rad/s. This may be verified by looking at the companion paper by Ulwick et al (including Kelley) published in the same issue as KH. There on page 6991 they state their sampling frequency to be \(~8000\) Hz, and on p. 6992 they say that their spectral analysis Nyquist frequency is above \( 4000 \) Hz. Further, they state that they therefore have a "spatial resolution corresponding to scale sizes of about 12 cm". Note that \( \ln y = \frac{V_r}{f_{ny}} \) gives 11.2 cm, and there is no \( 2\pi \) ambiguity in what is reported. This is verified on page 7004 of KU, where they state that the frequency, \( f \), of \( 250 \) Hz, corresponds to a wavelength, \( \lambda \), of \( 1.8 \) m and a wavenumber, \( k \), of \( 3.5 \) rad/m. It should also be noticed that the terms "scale size" and "wavelength" are used interchangeably in both the Ulwick et al. and the KU papers.

Looking at the Brattli paper, on page 7100 they state that their sampling frequency is \( 2441.4 \) Hz, giving "a spatial resolution of better than \(~0.5\) m". In the caption to their figure 1 they state that their rocket velocity, \( V_r = 560 \) m/s. Using the KU relationships given above and the fact that \( f_{ny} = \frac{f_{sample}}{2} \), one finds a Nyquist wavelength, equivalent to a "spatial resolution", of \( 0.4587 \) m. Again, there is no \( 2\pi \) ambiguity. In addition, the upper axis of their figure 1 shows their spectrum extending to \( 0.4587 \) m, and the lower axis extends to \( 13.696 \) rad/m, consistent with KU relationship \( k = (2\pi)/\lambda \). The only objection that I can see the referee having is that the wavenumber axis is labeled as "\( 1/m \)" instead of the more correct "\( \text{rad/m} \)" and that the upper axis is labeled "Scale size" rather than wavelength. I would point out that this mixture of the terms "wavelength" and "scale size" is entirely consistent with the KU paper.

Returning to the KU paper, they define in equation 3 the Kolmogorov "microscale", \( h \), as \( h = (n3/e)^{1/4} \). They then go on to state on page 2004 "that \( h \) actually is well within the viscous subrange and in fact occurs at a wavelength which is 7-12 times smaller than the so-called inner scale for turbulence, \( \lambda_0 \). The latter is defined as the intercept of the power law slopes in the inertial and viscous subranges". This terminology is exactly the same as that used by Brattli et al at the top of page 7105. In addition, KU quote a value...
for $\ell_0$ between 100 and 200 Hz (again not angular frequency) corresponding to 2.25 - 4.5 m. Clearly, this is consistent with calculating the wavenumber from $f = V_r/(2\pi k)$ (exactly as Brattli et al have done), and then calculating the wavelength, or as KU term it the "inner scale for turbulence", as $\ell_0 = (2\pi k)/(2\pi)$ (again in agreement with what Brattli et al have done and report). Finally, KU give equation 10 as $\ell_0 = (7-12)h$, and Brattli et al. give their equation 14 as $\ell_0 = 9.9(n3/e)^{1/4}$. Given the KU definition of $h$ in their equation 3, and the fact that 9.9 is, in fact, a number lying between 7 and 12, these analyses are identical and there is no missing factor of $2\pi$ as the referee states.

As final note, KU, near the top of page 7007, state that "$k_s = 2.09$ m$^{-1}$ is the wave number responsible for the back-scatter at 50 MHz." This suggests that the wavelength that will be preferentially scattered is $\lambda_r/2$, where $\lambda_r$ is the radar wavelength, and the corresponding wavenumber is $(2\pi)/(\lambda_r)$. In the crystallographic literature, these are referred to as the Bragg wavelength and wavenumber, respectively, as Bragg’s law is formulated in terms of wavelength. That is, given a lattice dimension of spacing $d$, Braggs law for the wavelength of preferential back-scatter is given by $n\lambda = 2d$, where $n$ is an integer. This law, in this form, can be recovered using the lattice wavenumber of $G$, as well.

So, I am not sure on what the referee is basing his or her objections. The only thing I can conclude is that it is based upon semantics, where the referee believes that the use of the word "scale" or "Bragg scale" automatically invokes a $1/k$ relationship. However, scale and wavelength have been used interchangeably in the very literature on which the referee is basing his or her objections.

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