Relevant Changes

In accordance with our replies to the referee comments, the following changes have been applied to manuscript and supplement. The changes to the manuscript are highlighted further below.

33497 | 4 Added:
“The mixed solution parameterization framework has been determined by a multi-functional fitting, limited to one parameter per compound, to match the results of ISORROPIA II. Subsequently, the multi-phase and multi-component thermodynamic system has an analytical solution by using our consistent set of analytical equations together with the vi-coefficient of M2012. Therefore, our mixed solution parameterization framework has similar applicability as ISORROPIA II.”

33500 | 5-6 Changed:
“subsequent reactions” to “the reactions”

33504 | 21 Changed:
“… concentrations constant, which we directly obtain from the NRO (see Sect. 2.5)” to “… concentrations constant. Since we do not use at this computation step any dissociation constant, we obtain the maximum concentrations (NRO, max) directly obtain from the NRO.”

33507 | 9 Changed:
“Despite the large differences” to “Despite fundamental differences”

33510 | 19 Changed:
“we compute the RHDMIN from Eq. (5b) of M2012” to “we compute the RHDMIN from Eq. (5b) of M2012 (or Eq. A6 of the Appendix).”

33512 | 8 Added:
“See Fig. S2.2 for a flowchart of the Mixed solution RHD calculation.”
New Figure S2.2 added to the supplement:
“S2.2: Mixed solution RHD calculation (B9 of the EQSAM4clim flowchart, Fig. S2.1).”

33512 | 14 Changed:
“(partial) aerosol water” to: “partial aerosol water”

33513 | 20 Added:
“Note that Eq.(22) can be equally used for so-called metastable aerosols, for which the formation of solid salts is generally not considered.”

33513 | 21 Added:
“We evaluate our parameterization” to “We apply our parameterization”
gas-liquid-solid-partitioning” to “gas-liquid-solid partitioning”

“we refer to this article” to “we refer to Metzger et al. (2006)”

“(see also the comparison in the Supplement, Sect. S3.4).”

(see Table 5 for the statistics and the Appendix for the evaluation metrics).”

“extent” to “extend”

“Besides significant computational speed-up, another advantage is that our framework minimizes
the number of thermodynamic data that are typically required, while it enables greater
flexibility with respect to the extension to other compounds, not considered in this evaluation.”

“sneak preview. Figure S2” to “sample. Figure S2.1”

Appendix C: Evaluation metrics

Table 1. Added the definition of terms in the left-most column to the Table caption.

Table 5. MINOS aerosol statistics (Fig.8-9): (top) fine mode, (bottom) coarse mode.
EQSAM4clim (EQ4c) and ISORROPIA II (ISO2) versus MINOS observations (Aug 2001).”

“Note that at zero ammonia, H2SO4 is at a maximum; shown in Fig. S4.”

EQUISOLV II Comparison – case 16. Bulk aerosol water mass as a function of RH for different sulfate molar ratios, fixed for the entire RH range (at constant T = 298.15 K).”

“20 Cases Comparison” to “EQUISOLV II Comparison”
Aerosol water parameterization: a single parameter framework

S. Metzger\textsuperscript{1,2}, B. Steil\textsuperscript{2}, M. Abdelkader\textsuperscript{1}, K. Klingmüller\textsuperscript{2}, L. Xu\textsuperscript{3}, J. E. Penner\textsuperscript{4}, C. Fountoukis\textsuperscript{5,1}, A. Nenes\textsuperscript{6,5,7}, and J. Lelieveld\textsuperscript{1,2}

\textsuperscript{1}The Cyprus Institute, Nicosia, Cyprus
\textsuperscript{2}Max Planck Institute For Chemistry, Mainz, Germany
\textsuperscript{3}Scripps Institution of Oceanography, San Diego, USA
\textsuperscript{4}University of Michigan, Ann Arbor, Michigan, USA
\textsuperscript{5}Institute of Chemical Engineering Sciences, Foundation for Research and Technology Hellas, Patras, Greece
\textsuperscript{6}Schools of Earth and Atmospheric Sciences and Chemical and Biomolecular Engineering, Georgia Institute of Technology, Atlanta, GA, USA
\textsuperscript{7}Institute of Environmental Research and Sustainable Development, National Observatory of Athens, Palea Penteli, Greece

Correspondence to: S. Metzger (s.metzger@cyi.ac.cy, swen.metzger@mpic.de)
Abstract

We introduce a framework to efficiently parameterize the aerosol water uptake for mixtures of semi-volatile and non-volatile compounds, based on the coefficient, \( \nu_i \). This solute specific coefficient was introduced in Metzger et al. (2012) to accurately parameterize the single solution hygroscopic growth, considering the Kelvin effect – accounting for the water uptake of concentrated nanometer sized particles up to dilute solutions, i.e., from the compounds relative humidity of deliquescence (RHD) up to supersaturation (Köhler-theory). Here we extend the \( \nu_i \)-parameterization from single to mixed solutions. We evaluate our framework at various levels of complexity, by considering the full gas-liquid-solid partitioning for a comprehensive comparison with reference calculations using the E-AIM, EQUISOLV II, ISORROPIA II models as well as textbook examples. We apply our parameterization in EQSAM4clim, the EQuilibrium Simplified Aerosol Model V4 for climate simulations, implemented in a box model and in the global chemistry-climate model EMAC. Our results show: (i) that the \( \nu_i \)-approach enables to analytically solve the entire gas-liquid-solid partitioning and the mixed solution water uptake with sufficient accuracy, (ii) that, e.g., pure ammonium nitrate and mixed ammonium nitrate – ammonium sulfate mixtures can be solved with a simple method, and (iii) that the aerosol optical depth (AOD) simulations are in close agreement with remote sensing observations for the year 2005. Long-term evaluation of the EMAC results based on EQSAM4clim and ISORROPIA II will be presented separately.

1 Introduction

The most comprehensive description of aerosol composition and hygroscopic growth is provided by models that calculate the full gas-liquid-solid partitioning, i.e., the composition and state of the ion-pairs over the wide range of temperatures and relative humidities from the surface in the tropics to the winter polar stratosphere. Since thermodynamic equilibrium is the final state of kinetic processes, many modeling approaches assume equilibrium, which
is reasonable if the atmospheric processes that lead toward it are fast compared to those
that lead away from it (Wexler and Potukuchi, 1998).

To calculate the multiphase partitioning, composition and associated water uptake of mul-
ticomponent atmospheric aerosols, various equilibrium models (EQMs) have been devel-
oped over the past decades including: EQUIL (Bassett and Seinfeld, 1983), KEQUIL (Bas-
sett and Seinfeld, 1984), MARS (Saxena et al., 1986), MARS-A (Binkowski and Shankar,
1995), SEQUILIB (Pilinis and Seinfeld, 1987), AIM (Wexler and Seinfeld, 1991), SCAPE
(Kim et al., 1993a, b; Kim and Seinfeld, 1995), SCAPE2 (Meng et al., 1995), EQUISOLV
(Jacobson et al., 1996), ISORROPIA (Nenes et al., 1998, 1999), EQUISOLV II (Jacob-
son, 1999), GFMN (Ansari and Pandis, 1999, 2000), EQSAM (Metzger et al., 2002a, b),
AIM2 and E-AIM (Wexler and Clegg, 2002), HETV (Makar et al., 2003), ADDEM (Topp-
ing et al., 2005a, b), MESA (Zaveri, 2005), EQSAM2 (Metzger et al., 2006), UHAERO
(Amundson et al., 2006), ISORROPIA II (Fountoukis and Nenes, 2007), EQSAM3 (Metzger
and Lelieveld, 2007), UCD (Zhang and Wexler, 2008), AIOMFAC (Zuend et al., 2011) and
EQSAM4clim (this work).

These EQMs are often embedded in aerosol dynamical models (e.g., Pilinis et al., 2000),
but the gas-aerosol partitioning and especially the associated water uptake controls the
aerosol size distribution, if treated explicitly, which makes the development of EQMs most
critical. As a consequence, EQMs vary in the degree of complexity and computational ap-
proaches, while almost all EQMs are computationally expensive, due to the complexity of
the underlying multicomponent and multiphase thermodynamics. Either numerical accuracy
has higher priority than computational efficiency (i.e., iterations to reach equilibrium are ex-
tensive), or the computational approaches are comprehensive. Often both applies. Despite
the large efforts, computational efficiency, especially if combined with accuracy and flex-
ibility regarding the number of chemical compounds that can be considered, remains to
be a challenge, which is especially relevant for global atmospheric aerosol-chemistry and
climate modeling.

To meet this challenge we introduce in Sect. 2 an unique single parameter framework,
which is subsequently applied in Sect. 3. Our framework allows to efficiently parameterize
the aerosol water uptake for mixtures of semi-volatile and non-volatile compounds, being entirely based on the single solute specific coefficient introduced in Metzger et al. (2012). Additional results and textbook examples of Seinfeld and Pandis (2006) are presented in the Supplement. We conclude with Sect. 4.

2 Mixed solution parameterization framework

We introduce a mixed solution parameterization framework to efficiently calculate the aerosol water uptake for mixtures of semi-volatile and non-volatile compounds with the constraint of using only one parameter, i.e. $\nu_i$. The solute specific coefficient $\nu_i$ was introduced in Metzger et al. (2012) – referred to in the following as M2012 – to accurately parameterize the single solution hygroscopic growth, also considering the Kelvin effect. M2012 have shown that the $\nu_i$-approach is valid for a wide range of atmospheric conditions. To investigate the potential of the $\nu_i$-approach with respect to mixtures of salt compounds, we extend in this work the $\nu_i$-parameterizations from single to mixed solutions. Since computational efficiency is a requirement for our parameterization framework, we minimize the overall computational burden by a set of key-constraints:

1. Solving the multicomponent and multiphase partitioning analytically, by using a consistent set of equations, based on one compound specific single solute coefficient, $\nu_i [\cdot]$. This set of equations includes the solute molality, $\mu_s \text{[mol(solute) kg}^{-1}(\text{H}_2\text{O})\text{]},$ and its equivalent expression in terms of the mass fraction solubility, $\chi_s [\cdot]$. Both are the essential thermodynamic properties in our framework and only depend on $\nu_i$; the temperature ($T$), relative humidity (RH) and the particle dry diameter ($D_s$) are given;

2. Breaking down the complexity of aerosol thermodynamics as much as possible, without the loss of crucial information and critical numerical accuracy, by using chemical domains with a neutralization order for all salt compounds listed in Table 1;

3. Minimizing the dependencies on the required thermodynamic data by using a predetermined $\nu_i$-coefficient for each electrolyte listed in Table 1;
4. Assuming \( \nu_i \), constant for the entire range of water activity, \( a_w \).

The relevant single solute equations (of M2012) are summarized in Appendix A. The mixed solution parameterization framework has been determined by a multi-functional fitting, limited to one parameter per compound, to match the results of ISORROPIA II. The multi-phase and multi-component thermodynamic system has an analytical solution when our consistent set of analytical equations and the \( \nu_i \)-coefficient of M2012 are used. Consequently, our parameterization framework has a similar applicability as ISORROPIA II.

### 2.1 Pre-determined \( \nu_i \)

M2012 have detailed that a (unitless) single solute coefficient, i.e., \( \nu_i \) [–], can be accurately deduced from one reference data-pair of solute molality, \( \mu_s \), and the corresponding water activity, \( a_w \) [–]. We use a data-pair at solute saturation to pre-determine \( \nu_i \), since measurements are available for all major salt compounds that are of interest in atmospheric aerosol modeling. For the salt compounds listed in Table 1 we use the mass fraction solubility, \( w_s \) [%], which is an equivalent expression to the saturation molality, \( \mu_s,\text{sat} \). For the corresponding \( a_w \), we use the available Relative Humidity of Deliquescence (RHD) values and obtain \( \nu_i \) by solving Eq. (5b) of M2012 with a root finding method (bisection). To be consistent with ISORROPIA II we determine here \( \nu_i \) from \( w_s \) and RHD data at temperature \( T_o = 298 \) [K]. The water activity data used by ISORROPIA II (and other EQMs), are tabulated only for room temperature. We therefore do not consider the \( T \)-dependency of \( \nu_i \) in this work.

Table 1 lists the pre-calculated \( \nu_i \) values for each salt compound considered, together with the required thermodynamic data: Stoichiometric coefficient \( \nu_s \) [–], the ion-pair charge \( Z_s \) [–], the single solute parameter \( \nu_i \) [–], the mass fraction solubility in percent \( W_s \) [%] \((w_s = W_s/100), the molar masses \( M_s \) [kg mol\(^{-1}\)], the densities \( D_s \) [kg m\(^{-3}\)], RHD\((T_o) \) [–] at reference temperature \( T_o = 298.15 \) [K], and the corresponding temperature coefficients,

\( T_{\text{coeff(RHD)}} [-] \). The RHD values are taken from Fountoukis and Nenes (2007); the other values of Table 1 are taken from the Handbook of Chemistry and Physics (Lide, 2005).

### 2.2 Chemical domains

To break down the complexity of aerosol thermodynamics as much as possible, we minimize the number of chemical compounds and equilibrium reactions that have to be considered. Following the original EQSAM approach (Metzger et al., 2002a), we define chemical domains with a sub-set of neutralization reactions that are considered for a given \( T \), RH and input concentrations (total of aerosol cations/anions and precursor gases); with all concentration units in \([\text{mol m}^{-3}(\text{air})]\). Our domain definition is listed in Table 2 and applied in our mixed solution framework with if-else logic and top–down approach. The potential aerosol neutralization levels depend on the input concentration ratio of total cations, \( t\text{CAT} \). The cations are balanced against the total sulfate anions, for which we consider for sulfate rich cases the total sulfates as bi-sulfate, \( t\text{HSO}_4^- \), or, for sulfate poor cases as total sulfate, \( t\text{SO}_4^{2-} \). In any case, these totals “\( t \)” need to exceed a threshold, \( \text{MIN} = 1 \times 10^{-15} [\text{mol m}^{-3}(\text{air})] \); below the computations are neglected for a given domain. Our definition of totals is given by our domain classification (Table 2), implicitly taking into account the maximum neutralization level that is theoretically possible for each domain:

- \( t\text{SO}_4 = \sum (1 \cdot \text{HSO}_4^- + 2 \cdot \text{SO}_4^{2-}) \)
- \( t\text{HSO}_4 = \sum (1 \cdot \text{HSO}_4^- + 1 \cdot \text{SO}_4^{2-}) \)
- \( t\text{CAT} = \sum (2 \cdot \text{Ca}^{2+} + 2 \cdot \text{Mg}^{2+} + 1 \cdot \text{Na}^+ + 1 \cdot \text{K}^+ + 1 \cdot \text{NH}_4^+) \)

The domain definition (Table 2) is subsequently used to define the neutralization reaction order (see Sect. 2.3). The two semi-volatile compounds listed in Table 1, \( \text{NH}_4\text{NO}_3 \) and \( \text{NH}_4\text{Cl} \), are considered in our approach only in the sulfate neutral (D1) domain. On the other hand, bi-sulfate is taken into account only for the sulfate rich (D2) and very rich (D3) cases, while free sulfuric acid is considered only for the unneutralized sulfate case (D4).
2.3 Domain dependent neutralization reaction order

To avoid the numerical minimization of the Gibbs free energy, which is required to obtain the equilibrium composition of mixed solutions (Seinfeld and Pandis, 2006), we define for each domain (Table 2) a neutralization reaction order (NRO), which can practically be considered as the salting-out effect of salt solutes (Metzger and Lelieveld, 2007). For this work, we rank the cations and anions according to their preferred neutralization reaction by:

- Anions: $\text{SO}_4^{2-}$, $\text{HSO}_4^-$, $\text{NO}_3^-$, $\text{Cl}^-$
- Cations: $\text{Ca}^{2+}$, $\text{Mg}^{2+}$, $\text{K}^+$, $\text{Na}^+$, $\text{NH}_4^+$, $\text{H}^+$

which yields the NRO listed in Table 3. The ordering is based on numerous modeling studies, both extensive box-modeling comparisons (Metzger et al., 2002a, 2006, 2012) and global applications (Metzger et al., 1999, 2002b; Metzger and Lelieveld, 2007). Note that we have constrained the ordering for this work to achieve the closest agreement with ISORROPIA II for two reasons: (1) ISORROPIA II is currently the only EQM that is widely applied in global modeling; (2) and it also considers the mineral cations $\text{Ca}^{2+}$, $\text{Mg}^{2+}$, $K^+$.

To solve the mixed solution framework we apply the NRO to balance cation-anion pairs that have a non-zero ion–ion product. Within a chemical domain (Table 2), the electrolyte compounds listed in Table 3 are subsequently formed for non-zero ion–ion product, until all cation-anion pairs are paired, or either all cations or anions are fully neutralized. To analytically solve the entire gas-liquid-solid partitioning, we consider at this stage all electrolytes in solution (computing totals of gas and ions). The gas-solid and gas-liquid partitioning of semi-volatile compounds, the liquid-solid partitioning and the water uptake are determined in that order in subsequent and independent computational steps (Sect. S2 in the Supplement).

2.4 Treatment of semi-volatile compounds

Table 1 includes two semi-volatile compounds that exhibit the gas-liquid-solid partitioning, i.e., ammonium nitrate, $\text{NH}_4\text{NO}_3$, and ammonium chloride, $\text{NH}_4\text{Cl}$. Both are allowed in our
framework only in D1, provided that a surplus ammonium, NH$_4^+$, is available. Our implicit assumption is that all sulfates are neutralized first through subsequent reactions with cations (Sect. 2.3). Only excess ammonium may further neutralize anions, nitrate, NO$_3^-$, and/or chloride, Cl$^-$. Thus, semi-volatile compounds can only partition into the particles, if the concentration product exceeds a threshold that is given by the temperature and humidity dependent equilibrium dissociation constant, $K_p(T, \text{RH})$. The equilibrium partitioning is detailed in Seinfeld and Pandis (2006) – in the following SP2006 (Sect. 10.4.3 ff).

2.4.1 RH < RHD – pure and mixed compounds

When the RH is below the RHD and the partial pressure product of gaseous (g) ammonia, NH$_3$(g), and nitric acid, HNO$_3$(g), with units either in [ppbv] or [mol m$^{-3}$(air)], equals or exceeds temperature dependent equilibrium dissociation constant, $K_p(T)$, solid (s) ammonium nitrate (AN), NH$_4$NO$_3$(s), is assumed to be formed instantaneously:

$$\text{NH}_3(g) + \text{HNO}_3(g) \leftrightarrow \text{NH}_4\text{NO}_3(s) \quad \text{(R1)}$$

For Reaction (R1) the equilibrium concentration of solid ammonium nitrate can be analytically computed from the gaseous concentrations by solving a quadratic equation – for our examples in the Supplement we use [ppbv]. We compute the temperature dependency of the equilibrium dissociation constants, $K_p(T)$, following (Fountoukis and Nenes, 2007):

$$K_p(T) = K_p^o \times \exp[a \times (T_o/T - 1) + b \times (1 + \ln(T_o/T) - T_o/T)]$$

with $T$ and $T_o = 298.15$ K, the ambient temperature and reference temperature, respectively. The $K_p^o(T_o)$ values are given in Table 4 in [ppbv$^2$] for $T_o$ and reference pressure, $P_o = 1$ [atm] = 101325 [Pa], together with the dimensionless temperature coefficients, $a$ and $b$ [-]. For applications on a mole basis (e.g., for EQSAM4clim), $K_p(T)$ [ppbv$^2$] can be converted to $[(\text{mol m}^{-3}(\text{air}))^2]$, using $K_{p,\text{mol}}(T) = K_p(T)/(R/P \times T)^2$. 


with $R = 8.314409 \text{[Pa m}^3\text{mol}^{-1} \text{K}^{-1}]$ the gas constant and $P = 101325 \text{[Pa]}$ ($R/P_o \times T_o = 24.465 \text{[L mol}^{-1}]$).

The equilibrium dissociation constant of $\text{NH}_4\text{NO}_3$ is sensitive to temperature changes and varies over more than two orders of magnitude for typical ambient conditions. This is illustrated in Fig. 10.19 of SP2006 to which we refer for a detailed discussion. For comparison, Fig. 1 shows the same $K_{\text{p,AN}}$ values as a function of $T$ at $\text{RH} < \text{RHD}$ for the EQSAM4clim and ISORROPIA II applications. Although the results are similar, those of SP2006 are slightly lower since their values are obtained from a slightly different equation, i.e., $K_{\text{p,AN}}(T) = \exp(84.6 - 24.220/T - 6.1 \times \ln(T/T_o))$ – see Eq. (10.91) of SP2006 (respectively Eq. (9.91) and Fig. 9.19 of Seinfeld and Pandis, 1998). Note that Reaction (R1) applies for the gas-aerosol partitioning over dry aerosols – pure $\text{NH}_4\text{NO}_3(s)$, or any mixture of $\text{NH}_4\text{NO}_3(s)$ with other dry salt-compounds. An example is given in Sect. S1.1 in the Supplement.

2.4.2 RH ≥ RHD – pure compound

For the wet case, with RH above the compound RHD or mixed solution RHD (see below), the situation is more complicated. In contrast to the gas-solid partitioning described above, the gas-liquid equilibrium partitioning of, e.g., gaseous ammonia, $\text{NH}_3(g)$, and nitric acid, $\text{HNO}_3(g)$, is in equilibrium with aqueous ammonium nitrate, $\text{NH}_4\text{NO}_3(aq)$, when the vapor pressure product of the gases exceeds its temperature and humidity dependent equilibrium dissociation constant, $K_{\text{p,AN}}(T, \text{RH})$. The salt compound formed is – when equilibrium is reached – additionally dissociated into a cation $\text{NH}_4^+(aq)$ and anion $\text{NO}_3^-(aq)$ pair.

1. Following SP2006 (their Sect. 10.4.3) Reaction (R1) expands to:

$$\text{NH}_3(g) + \text{HNO}_3(g) \leftrightarrow \text{NH}_4^+(aq) + \text{NO}_3^-(aq)$$

EQ: $[\text{NH}_3(g)] \times [\text{HNO}_3(g)] = K_{\text{p,AN}}(T, \text{RH}) = \frac{\gamma_{\text{AN}}^2 \times \mu_{\text{NH}_4^+(aq)} \times \mu_{\text{NO}_3^-(aq)}}{K_{\text{AN}}}$

9
For Reaction (R2) the equilibrium partitioning dissociation constant is now a function of \( T \) and RH. In the notation of SP2006 (see their Eq. 10.99), \( K_{p,AN}(T, RH) \) is related to the temperature dependent equilibrium constant \( k_{AN} \). \( k_{AN} \) [mol\(^2\) kg\(^{-2}\) atm\(^{-2}\)], depends on the ion molalities [mol kg\(^{-1}\)(H\(_2\)O)] of ammonium and nitrate, i.e., \( \mu_{\text{NH}_4^+}^{\text{(aq)}} \) and \( \mu_{\text{NO}_3^-}^{\text{(aq)}} \), and on the corresponding mean molal binary activity coefficient of aqueous ammonium nitrate, i.e., \( \gamma_{AN}^2 \); squared because of the cation–anion product. Solving Reaction (R2) requires iterations. To determine the aqueous phase concentration of all compounds that can exist in solution at given \( T \) and RH requires knowledge of the total aerosol water mass (see below), which in turn depends on the solute concentrations and according to Reaction (R2) on activity coefficients. Thus, since \( \gamma_{AN} \) is a function of the aqueous phase concentration, \( K_{p,AN}(T, RH) \) has no analytical solution. According to the thermodynamic literature, the standard treatment is therefore quite comprehensive and requires complex thermodynamic codes.

2. Here we express the product \( \left( \gamma_{AN}^2 \times \mu_{\text{NH}_4^+}^{\text{(aq)}} \times \mu_{\text{NO}_3^-}^{\text{(aq)}} \right) \) of Reaction (R2) to be only a function of \( \nu_i \) and RH, which is motivated by M2012, since their \( \mu_s \) is only a function of \( \nu_i \) and RH. To be able to solve Reaction (R2) analytically, we parameterize \( K_{p,AN}(T, RH) \) by introducing a solute specific correction term for Reaction (R1) which only depends on RH:

\[
K_p(T, RH) = K_p(T) \times \text{COEF}(RH)
\]  

(2)

At given \( T \) and RH, \( K_p(T, RH) \) is then a priori known, if COEF(RH) is independent of the solute concentration and associated water mass. This can be achieved either by fitting data (Metzger et al., 1999, 2002a; Hauglustaine et al., 2014), or expressing COEF(RH) in terms of the \( \nu_i \) and the RH-dependent solute molality parametrization of M2012; using their Eq. (5a). Utilizing further the relation Eq. (A11) of M2012, we can express the solute molality in terms of the solute mass fraction, \( \chi_s \), and define COEF(RH) for pure NH\(_4\)NO\(_3\)(aq) solutions in terms of \( \chi_s(RH) \), i.e., for \( RH \geq \text{RHD} \):

\[
\text{COEF}(RH) := 2 \times \chi_s^2(RH)
\]

(3)
with \( \text{COEF}(\text{RH}) := 1 \) for \( \text{RH} < \text{RHD} \). Equation (3) has been empirically derived to approximate the results obtained by ISORROPIA II (see Sect. 3). \( \chi_s(\text{RH}) \) denotes the solute mass fraction, which requires in our mixed solution parameterization framework only knowledge of \( \text{RH} \) and the solute specific coefficient, \( \nu_i \). In accord with the dry case (Reaction R1), aqueous solutions (Reaction R2) can now be analogously solved. Using Eqs. (2) and (3) to obtain \( K_{p,AN}(T, \text{RH}) \) at a given \( T \) and \( \text{RH} \), the quadratic equation, which has an analytical solution for the dry case, can now also applied to pure \( \text{NH}_4\text{NO}_3(\text{aq}) \) solutions. The direct solution of Reaction (R2) by using Eqs. (2) and (3) is exemplified in the Supplement.

The \( T \)-dependent equilibrium dissociation constant of \( \text{NH}_4\text{NO}_3 \), shown in Fig. 1, is also sensitive to changes in relative humidity and varies over orders of magnitude for typical ambient conditions. This is illustrated in Fig. 10.21 of SP2006 to which we refer for a detailed discussion. For comparison, Fig. 2 extends Fig. 1 showing the \( K_{p,AN}(T, \text{RH}) \) values as a function of \( \text{RH} \) at \( T = 298.15 \text{[K]} \) for the EQSAM4clim and ISORROPIA II applications. The line-points, which refer to pure \( \text{NH}_4\text{NO}_3(\text{aq}) \) solutions, are relatively close for the EQSAM4clim and ISORROPIA II results, but both are (for \( \text{RH} \leq \text{RHD} \)) roughly a factor two higher than the corresponding values of SP2006 (see Fig. 1); the constant \( K_{p,AN}(T) \) of SP2006 is included for reference (at \( T = 298.15 \text{[K]} \)). Note that with Eq. (2) and the quadratic form of Eq. (3) we can analytically approximate the solution of \( K_{p,AN}(T, \text{RH}) \) for \( Y = 1.0 \) (Fig. 2).

### 2.4.3 RH \( \geq \) RHD – mixed compound

According to SP2006 (and references therein), Reaction (R2) needs to be extended for mixed aqueous solutions to include an ionic strength factor.

1. Following the notation of SP2006 (see their Eq. 10.100), the equilibrium concentration (either in [ppbv] or [mol m\(^{-3}\)(air)]) of [\( \text{NH}_4\text{NO}_3 \)] in mixed aqueous solutions is
controlled by the presence of ammonium sulfate, \([(NH_4)_2SO_4]\), and depends on a dimensionless ionic strength factor \(Y\), which is defined by the ratio:

\[
Y := \frac{[NH_4NO_3]}{[NH_4NO_3] + 3 \times [(NH_4)_2SO_4]} \tag{4}
\]

To extend the calculation of the \(T\) and RH-dependent equilibrium dissociation constant to the case of multicomponent aqueous solutions of \(NH_4NO_3\), shown in Fig. 2, Eq. (4) needs to be considered such that \(K_{p,AN}(T, RH)\) becomes \(K_{p,AN}(T, RH, Y)\).

2. To satisfy our key-constraint (see Sect. 2) we avoid iterations in our mixed solution parameterization. We therefore parameterize \(K_{p,AN}(T, RH, Y)\) by expanding Eq. (2) to be additionally a function of \(Y\) (Eq. 4):

\[
K_p(T, RH, Y) = K_p(T) \times \text{COEF(RH, Y)} \tag{5}
\]

For Eq. (5), \(\text{COEF(RH, Y)} := \text{COEF(RH)} \times Y^{-0.8}\), where \(\text{COEF(RH)}\) is given by Eq. (3) and \(Y\) by Eq. (4), for which we use the concentration given by the NRO (Sect. 2.3), i.e., \(Y := \frac{[NH_4NO_3]_{(nro, max)}}{[NH_4NO_3]_{(nro)} + 3 \times [(NH_4)_2SO_4]_{(nro)}}\). The \(Y^{-0.8}\)-term has been empirically determined to approximate the results of ISORROPIA II by keeping the initial \(NH_4NO_3\) and \((NH_4)_2SO_4\) concentrations constant, which we directly obtain. Since we do not use at this computation step any dissociation constant, we obtain the maximum concentrations \((NRO, max)\) directly from the NRO (see Sect. 2.5). Equation (5) and the quadratic equation can be solved non-iteratively. The solution is detailed below (Sect. 2.5); examples are given in Sect. S1 in the Supplement.

Figure 2 shows that the results of EQSAMclim and ISORROPIA II exhibit a similar dependency on \(Y\) for \(K_{p,AN}(T, RH, Y)\), where the values decrease with decreasing \(Y\) according to the results and the discussion of SP2006 (see their Fig. 10.21). \(K_{p,AN}(T, RH, Y)\) is given here by the product of the gaseous concentrations of ammonia, \([NH_3]_{(g,AN)}\), and
nitric acid, \([\text{HNO}_3]_{(g,AN)}\), which are in equilibrium with either the solid \([\text{NH}_4\text{NO}_3]_{(s)}\) concentration, if \(\text{RH} < \text{RHD}\), or in equilibrium with the aqueous \([\text{NH}_4\text{NO}_3]_{(aq)}\) concentration when \(\text{RH} \geq \text{RHD}\) in case of pure \([\text{NH}_4\text{NO}_3]\) (zero \([(\text{NH}_4)_2\text{SO}_4]\), where \(Y = 1\)). Below the RHD, \(K_p(T, \text{RH}, Y)\) reduces to \(K_p(T)\) as given by Eq. (1) in Reaction (R1). Differences, which occur mainly in the mixed deliquescence humidity range, are discussed below (Sect. 2.6).

### 2.5 Solving \(\text{NH}_4\text{NO}_3/\text{NH}_4\text{Cl}\)-thermodynamic equilibrium – this work

To analytically compute the equilibrium concentrations of the two semi-volatile compounds, \(\text{NH}_4\text{NO}_3\) and \(\text{NH}_4\text{Cl}\), for a given \(\text{RH}\) and \(T\) with our mixed solution parameterization, we first solve all neutralization reactions at once for the domain by using the NRO (Table 3, Sect. 2.3) and the totals (gas + aerosol) of the cation and anion input concentrations. Thus we obtain the free ammonium \(\text{TA} = [\text{NH}_4^+]_{(nro,\text{free})}\) and nitrate \(\text{TN} = [\text{NO}_3^-]_{(nro,\text{free})}\), after all higher ranked cation-anion paris are paired. To enable a non-iterative solution, we do not use at this computation step any dissociation constant, so that we directly obtain from \(\text{TA}\) and \(\text{TN}\) the corresponding maximum ammonium nitrate concentration \([\text{NH}_4\text{NO}_3]_{(nro,\text{max})}\) = MIN([\(\text{TA}\), [\(\text{TN}\)]], which is possible for \(K_p(T, \text{RH}, Y) = 1\) and the given input concentration, \(T\) and \(\text{RH}\). Analogously, we compute the maximum ammonium chloride concentration from the final free ammonium and free chloride, \([\text{TC}] = [\text{Cl}^-]_{(nro,\text{free})}\), with \([\text{NH}_4\text{Cl}]_{(nro,\text{max})}\) = MIN(\(\text{TA}\), \(\text{TC}\)).

With the initial (maximum) values of \([\text{NH}_4\text{NO}_3]_{(nro,\text{max})}\) and \([(\text{NH}_4)_2\text{SO}_4]_{(nro,\text{max})}\) we can further solve Eq. (4). To obtain the final equilibrium concentrations, we compute the evaporative loss. For \([\text{NH}_4\text{NO}_3]_{(nro)}\), we compute the gaseous ammonia, \([\text{NH}_3]_{(g,AN)}\), and nitric acid, \([\text{HNO}_3]_{(g,AN)}\) from \([\text{NH}_4\text{NO}_3]_{(nro,\text{max})}\):

\[
\begin{align*}
[\text{TN}] &= [\text{NO}_3^-]_{(nro,\text{free})} \\
[\text{TA}] &= [\text{NH}_4^+]_{(nro,\text{free})} \\
[X] &= \frac{1}{2} \times \left( -([\text{TA}] + [\text{TN}]) + \sqrt{([\text{TA}] + [\text{TN}])^2 + 4 \times K_p(T, \text{RH}, Y)/(RT)^2} \right)
\end{align*}
\]
where the variable \([X]\) is used to obtain:

\[
[NH_3]_{(g,AN)} = [HNO_3]_{(g,AN)} = \text{MIN}([NH_4NO_3]_{(nro,max)}, [X])
\]  

With Eq. (9) we compute the final gaseous concentrations of HNO\(_3\)(\(g\)) and NH\(_3\)(\(g\)) from:

\[
[NH_3]_{(g)} = [TA] + [NH_3]_{(g,AN)} \tag{10}
\]

\[
[HNO_3]_{(g)} = [TN] + [HNO_3]_{(g,AN)} \tag{11}
\]

and the final ammonium nitrate equilibrium concentration from:

\[
[NH_4NO_3]_{(nro)} = [NH_4NO_3]_{(nro,max)} - [HNO_3]_{(g,AN)} \tag{12}
\]

One can now solve with Eqs. (6)–(12) the quadratic equation for the dry, pure or mixed solution cases. But, in contrast to Seinfeld and Pandis (1998) (see their Eq. 9.103), we compute with Eq. (8) the evaporative losses of gaseous concentrations at equilibrium.

Figure 3 shows a comparison of idealized box model calculations of EQSAM4clim (see Appendix B) and ISORROPIA II (more comprehensive calculations are shown in Sect. 3). The upper panels show the gas-liquid-solid partitioning concentration of NH\(_4\)NO\(_3\) for a binary solution with a fixed concentration of \(1 \mu\text{mol m}^{-3}(\text{air})\) of pure NH\(_4\)NO\(_3\), while the lower panels show the same for a mixed solution with each \(1 \mu\text{mol m}^{-3}(\text{air})\) of NH\(_4\)NO\(_3\) and \((\text{NH}_4)_2\text{SO}_4\) (both at \(T = 298.15\text{ K}\)). The left panels show NH\(_4\)NO\(_3\), the right the corresponding total mass loading. To solve the gas-solid and gas-liquid partitioning we have used the \(\nu_i\) based framework (Sect. 2) for EQSAM4clim, and for ISORROPIA II the option to iteratively calculate activity coefficients. A detailed calculation for this example is given in the Supplement (Sect. S1). Despite the large fundamental differences in both approaches, the comparison of these results is satisfactory for mixed solute concentration from which the aerosol water mass is derived in a subsequent calculation step – for EQSAM4clim without iterations.
2.6 Mixed solution RHD

To calculate the liquid-solid partitioning, we follow (Fountoukis and Nenes, 2007) and consider a mutual deliquescence RH range. In our framework, it depends on a minimum and maximum threshold: RHDMIN and RHDMAX, which are defined below. When the RH is below RHDMIN the aerosol is considered to be dry, while for RH above RHDMAX the aerosol is considered wet with all ionic compounds dissolved. In between a mixture can exist, with some compounds dissolved while other compounds are precipitated from the solution.

For mixed solutions (two or more compounds and water), only the amount that exists for RH > RHDMIN is considered in solution and allowed to contribute to the mixed solution water uptake. Otherwise, the compounds are considered to be instantaneously solid and precipitated from the solution. For all non-precipitated compounds, a weighted solute concentration is computed from which subsequently all partial water masses are obtained. The sum of all partial water masses yields the total water for the given aerosol composition, size, T and RH (see Sect. 2.7).

However, comparing the water uptake calculation of EQSAM4clim with reference calculations of, e.g., ISORROPIA II and E-AIM, is somewhat precarious. The reason is that for mixed solutions the calculated water mass mainly depends on the threshold at which the mixture is considered to take up water. The assumptions made to define the mixed solution RHD, or the mutual deliquescence RH range, are generally a major source of uncertainty in modeling the aerosol associated water uptake. First we discuss the procedure of Fountoukis and Nenes (2007), and then we describe our single parameter approach.

1. For ISORROPIA II, if the RH is within a mutual deliquescence RH range, the so-called MDRH region, the solution is assumed to be the sum of two weighted solutions; a “dry solution” (considering a pure dry case) and a “saturated liquid” solution (considering a pure liquid case). Then a numerical solution needs to be found based on a weighting factor (WF) for the dry and liquid solution that could be present in the given sub-domain (solute composition). The weighting factors are obtained from (RHD – RH)/(RHD – MDRH) using prescribed MDRH values, which have been mea-
sured and tabulated for certain mixtures of salt solutes (see Fountoukis and Nenes, 2007). When the RH is below the MDRH, only a solid phase is possible. Otherwise a liquid and solid phase may coexist (with MDRH < RH < RHD). For the latter case the aqueous phase concentrations are determined by WF. The summation over all partial water masses then yields the total aerosol water mass. But the gas-liquid-solid partitioning is re-iterated until the solution converges and the concentrations do not change further (equilibrate). For details see Fountoukis and Nenes (2007).

2. Here we follow the idea of a weighted mixed solution approach of ISORROPIA II, but we approximately solve the liquid-solid partitioning by computing the weighting factor non-iteratively. We compute the liquid-solid partitioning after solving the NRO (Sect. 2.3) and the gas-liquid partitioning (Sect. 2.4). For each salt compound \( j \) we analytically obtain the solid concentration \( n_j(s) \) from its aqueous \( n_j(nro) \) concentration (determined in the previous computation steps), using analogously to Fountoukis and Nenes (2007) a mixed solution weighting factor, \( WF_{j,mix} \):

\[
n_j(s) = n_j(nro) \times WF_{j,mix}
\]

with

\[
n_j(aq) = n_j(nro) - n_j(s)
\]

where \( n_j(nro) \) denotes an aqueous concentration of, e.g., \([NH_4NO_3](nro)\) from Eq. (12).

\( WF_{j,mix} \) is defined for each compound (the \( j \)th salt solute in Table 3) by:

\[
WF_{j,mix} := \frac{RH_{DMAX_j} - RH}{RH_{DMAX_j} - RH_{DMIN}}
\]

with always a positive sign: RHDMIN \(< RH_{DMAX_j} \) and RH \( \leq RH_{DMAX_j} \). RHDMIN, RH_{DMAX_j} are defined below. For RH \( \geq RH_{DMAX_j} \), WF_{j,mix} = 0; RH \( \leq RH_{DMIN} \), WF_{j,mix} = 1. Note that we use a different notation of RHDMIN and RH_{DMAX_j} (instead of the MDRH and RHD used by Fountoukis and Nenes, 2007) to indicate that
we are using different values and underlying mixed solution calculations, which do not necessarily have to yield the same results despite our constraint that the overall liquid-solid partitioning aims to be comparable.

To solve the liquid-solid partitioning analytically, i.e., without iteration, we modify the approach of Fountoukis and Nenes (2007). Each binary concentration \( n_{j(nro)} \) is weighted by the total solute concentration, \( n_{s,\text{sum}(nro)} = \sum_{j=1,N_{\text{max}}} n_{j(nro)} \) with all concentration units in \([\text{mol m}^{-3}(\text{air})]\). But in our framework, \( n_{s,\text{sum}(nro)} \) is directly obtained from the sum of all single solute concentrations that are formed by solving the neutralization reaction order (Sect. 2.3). In case a semi-volatile compound has been initially neutralized, e.g. \([\text{NH}_4\text{NO}_3]_{(nro,\text{max})}\), we additionally solve Eqs. (6)–(12) to obtain \( n_{j(nro)} = [\text{NH}_4\text{NO}_3]_{(nro)} \) (Sect. 2.4), before we obtain a solute specific weighting factor, \( WF_j \), from:

\[
WF_j := \frac{n_{j(nro)}}{n_{s,\text{sum}(nro)}} = \frac{n_{j(nro)}}{\sum_{j=1,N_{\text{max}}} n_{j(nro)}}
\]  

(16)

The maximum value of \( N_{\text{max}} \) is limited by the domain dependent NRO (see Table 3). It refers to aqueous solutes at this stage. The liquid-solid partitioning is computed below. The concentration weighted maximum \( RHD_j \) (upper threshold), which normally needs to be computed iteratively for each compound, is here directly obtained using \( WF_j \):

\[
RHD_{\text{MAX}} = RHD_{\text{MIN}} \times WF_0^{0.25} + RHD \times (1 - WF_0^{0.25})
\]  

(17)

In case of mixed solutions, Eq. (17) is used to obtain the upper RH-threshold, otherwise the compound’s \( RHD_j \) given in Table 1 are used, while \( RHD_{\text{MIN}} \) is computed here directly for \( n_{s,\text{sum}(nro)} \) only from \( \nu_i \) (see below). \( WF_j \) is introduced here with an exponent (empirically derived) to parameterize the results of ISORROPIA II, which uses iterations to solve the liquid-solid partitioning using MDRH measurements as the lower RH-threshold in Eq. (17).
To adhere to our key-constraints (Sect. 2, i.e., to minimize the dependency on the required thermodynamic data) we compute the RHDMIN from Eq. (5b) of M2012 (or Eq. (A6) of the Appendix), by using the mixed solution values for \( \mu_s \) and \( \nu_i \) that correspond to \( n_{s,\text{sum}(\text{nro})} \) (Eq. 16). Assuming \( K_e = 1 \), \( A = 1 \) and \( B = 0 \), the single RHDMIN value can be obtained from:

\[
\text{RHDMIN} := \left( 1 + \mu_s^o \times M_w \times \nu_{i,mix} \times \left( \frac{1}{\mu_s^o \times \mu_{s,\text{sat},\text{mix}}} \right)^{\nu_{i,mix}} \right)^{-1}
\]  

(18)

\( \mu_{s,\text{sat},\text{mix}} \) is the saturation solute molality and \( \nu_{i,mix} \) the solute specific constant of the mixed solution. \( \mu_{s,\text{sat},\text{mix}} \) and \( \nu_{i,mix} \) are introduced here. \( \mu_s^o = 1 \) [mol kg\(^{-1}\)] is the reference molality to match units. \( M_w \) [kg mol\(^{-1}\)] is the molar mass of water.

Due to a lack of experimental data, we approximate \( \mu_{s,\text{sat},\text{mix}} \) from the summation over all single solute molalities, \( \mu_{s,\text{sat},\text{single}} \) [mol kg\(^{-1}\)], using the relation to the mass fraction solubility (see, e.g., Eq. A11 of M2012). We therefore obtain \( \mu_{s,\text{sat},\text{mix}} \) from:

\[
\mu_{s,\text{sat},\text{mix}} := \sum_{j=1,N_{\text{max}}} \left[ \frac{M_{j,\text{single}}}{M_{s,\text{mix}}} \right] \frac{1}{100/W_{j,\text{single}} - 1}
\]  

(19)

\( M_{j,\text{single}} \) [kg mol\(^{-1}\)] is the single solute molar mass, \( W_{j,\text{single}} \) [%] its mass fraction solubility. The data are given in Table 1 for all compounds considered in this work.

With the mixed solution molality, \( \mu_{s,\text{sat},\text{mix}} \), we can directly compute the mixed solution solubility, \( w_{s,\text{mix}} \), if we use as the corresponding total molar mass the sum of the molar masses over all \( (N_{\text{max}}) \) compounds that can dissolve in the mixed solution, i.e., \( M_{s,\text{mix}} = \sum_{j=1,N_{\text{max}}} M_{j,\text{single}} \) (same compounds and \( N_{\text{max}} \) as in Eq. 16):

\[
w_{s,\text{mix}} := \frac{1}{(\mu_{s,\text{sat},\text{mix}} \times M_{s,\text{mix}})^{-1} + 1}
\]  

(20)
with $0.1 < w_{s,\text{mix}} < 1$. Finally, we can obtain with Eq. (20) the solute specific constant that corresponds to the mixed solution, $\nu_{i,\text{mix}}$, using an empirical equation that approximates $\nu_{i,\text{mix}}$ from the corresponding mixed solution solubility $w_{s,\text{mix}}$:

$$\nu_{i,\text{mix}} := (0.25 \times \ln(w_{s,\text{mix}}) + 1)^{-1}$$

(21)

Thus, with Eq. (19) we solve Eq. (20) and with Eq. (20) we solve Eq. (21). With Eqs. (21) and (19) we solve Eq. (18) to obtain RHDMIN. And with Eq. (16) we solve Eq. (17) to obtain RHDMAX\(_j\). Together with RHDMIN we solve Eq. (15) to obtain \(WF_{j,\text{mix}}\). \(WF_{j,\text{mix}}\) is then used to compute the liquid-solid partitioning from Eqs. (13)–(14) after solving the NRO (Sect. 2.3) and the gas-liquid partitioning (Sect. 2.4), in case of semi-volatile compounds. Finally, the aerosol water uptake is computed for each salt compound that exists in the aqueous phase at the given \(T\) and \(RH\) from \(n_{j(aq)}\) (Eq. 14). \textbf{See Figure S2.2 in the Supplement for a flowchart of the calculation.}

### 2.7 Aerosol water uptake

To calculate the mixed solution aerosol water uptake, the standard procedure employs the widely used ZSR-mixing rule (see, e.g., SP2006, Eq. 10.98). Assuming that solute concentrations are in equilibrium with the ambient air, the total aerosol water mass, \(m_{w(\text{mix})}\) [kg m\(^{-3}\)(air)], can be directly obtained from the sum of all pure compound (partial–partial) aerosol water masses in case of a mixed solution (\(N\)-compounds dissolved):

$$m_{w,\text{mix}} = \sum_{j=1,N} m_{w,j} = \sum_{j=1,N} \frac{n_{j(aq)}}{\mu_{j(aq)}}$$

(22)

Here we follow the standard procedure, while the liquid-solid partitioning and the \(N\)-compounds in the aqueous phase are solved non-iteratively with Sect. 2.6. \(N\) can differ from \(N_{\text{max}}\) considered in Eq. (16), because certain salt solutes may precipitate from the mixed solution during the liquid-solid partitioning so that \(N \leq N_{\text{max}}\). With increasing RH (from RHDMIN up to RHDMAX\(_j\)) an increasing number of compounds is considered for the
water uptake calculations by Eq. (22). The partial aerosol water masses, \( m_{w,j} \), which are associated with each binary solution (one compound and water), \( n_{j(aq)} \) [mol m\(^{-3}\) (air)], can be directly obtained from tabulated single solute molalities, \( \mu_{j(aq)} \) [mol(solute) kg\(^{-1}\)(H\(_2\)O)] (see Appendix A1), or parameterized based on Eq. (5a) of M2012 (Appendix A2, Eq. A3).

In case the RH is below the \( T \)-dependent RHD, or the RHDMIN, we assume the compound to be dry and the partial aerosol water mass to be zero. Using the RH-dependent \( \mu_{j(aq)} \) parameterization of M2012 (their Eq. 5a), we can solve Eq. (22) without iterations.

Our mixed solution framework is independent of the total aerosol water mass, because:

1. \( n_{j(aq)} \) is independent of \( m_{w,mix} \), since it is directly given by our NRO (Sect. 2.3);
2. \( K_{p,AN}(T, RH, Y) \) is independent of \( m_{w,mix} \), because of our \( \chi_s(RH) \)-based parameterizations of semi-volatile compounds, i.e., the Eqs. (2)–(5) (Sect. 2.4);
3. \( \chi_s(RH) \) is independent of \( m_{w,mix} \), since it is directly given by \( \mu_{j(aq)}(RH) \) (depending also only on \( \nu_i \), and RH due to the relation Eq. A11 of M2012);
4. \( \mu_{j(aq)} \) is independent of \( m_{w,mix} \), since it only depends on \( \nu_i \), and RH
   \( (\mu_{j(aq)} \) is based on Eq. 5a of M2012 and included with \( \chi_s \) in Appendix A);
5. Mixed solution RHDMIN is independent of \( m_{w,mix} \), because of our mixed solution weighting factor, \( WF_{mix} \), parameterizations, which also only depend on \( \nu_i \), and RH;
6. \( m_{w,j} \) is independent of \( m_{w,mix} \), because of the independence of 1.–5.

Finally, Eq. (22) is solved for diagnostic output only, since \( m_{w,mix} \) does not need to be recalculated. Different from ISORROPIA II \( m_{w,j} \) and \( m_{w,mix} \) are not central in our gas-liquid-solid partitioning computations because of the \( \nu_i \)-concept, which requires that the thermodynamic key-properties, \( \mu_{j(aq)} \) and \( \chi_s \), only depend on RH and \( \nu_i \). Note that Eq. (22) can be equally used for so-called metastable aerosols, for which the formation of solid salts is generally not considered.
3 Application

We evaluate our parameterization using EQSAM4clim. EQSAM4clim is entirely based on the mixed solution framework described in Sect. 2, which builds on the $\nu_1$-approach of M2012. The underlying single solute parameterization of solute molality, $\mu_s$, and the relation to the solute mass fraction, $\chi_s$, are summarized in Appendix A. The Appendix also includes a short description of EQSAM4clim (Sect. B), while the computational algorithm of EQSAM4clim is detailed in the Supplement (Sect. S2).

To evaluate EQSAM4clim we compare the single solute and mixed solution aerosol water uptake, as well as various other aerosol properties, against different reference models using box and global modeling calculations at various levels of complexity:

1. Fixed solute concentrations (9 cases): ISORROPIA II and E-AIM (see also Sect. S3.1 in the Supplement)
2. Variable ammonia concentration: ISORROPIA II and SP2006 (see also Sect. S3.2 in the Supplement)
3. Variable solute concentrations (20 cases): ISORROPIA II and EQUISOLV II (see also Sect. S3.3 in the Supplement)
4. Field observations (MINOS campaign, 184 cases): ISORROPIA II (see also Sect. S3.4 in the Supplement)
5. EMAC chemistry-climate model (year 2005): ISORROPIA II

Selected results of each application case (1–5) are shown below, while the complete set of results are shown in the Supplement (Sect. S3). Throughout this work, all EQSAM4clim results will be primarily evaluated with respect to its ability to accurately simulate the water uptake of atmospheric aerosols, as this is a key process in climate modeling with our EMAC chemistry-climate model.
3.1 Fixed solute concentrations (9 cases): ISORROPIA II and E-AIM

Figure 4 shows the total aerosol water mass, $m_{w,mix}$ [kg m$^{-3}$ (air)] obtained by Eq. (22) for EQSAM4clim in comparison to the results of ISORROPIA II and E-AIM for the mixed solution case of NH$_4$NO$_3$ and (NH$_4$)$_2$SO$_4$ shown in Fig. 3 (lower panels). The results are based on the full gas-liquid-solid partitioning; for EQSAM4clim on Sect. 2. This first example considers the simplest calculation case: dry compound concentration fixed to 1 [µmol m$^{-3}$ (air)]. Figures 4 and S3 in the Supplement (see Supplement, Sect. 3.1) show that the relatively largest differences occur for all cases in the (mutual) deliquescence range, while the general water uptake above this range is very similar for all three equilibrium models, despite the fundamental differences in the underlying approaches of the thermodynamic models.

3.2 Variable NH$_3$ concentration: ISORROPIA II and SP2006

To further evaluate the aerosol water uptake calculations of EQSAM4clim for variable concentrations, we first compare the mixed solution composition of NH$_4$NO$_3$ and (NH$_4$)$_2$SO$_4$ (at $T = 298.15$ K) as a function of total ammonia (NH$_3 +$ NH$_4^+$), following SP2006; see their Fig. 10.23. Figure 5 shows the corresponding results in [µg m$^{-3}$ (air)] of EQSAM4clim and ISORROPIA II – (from top to bottom) for: ammonium, nitrate, sulfate and aerosol water, with bi-sulfate, sulfuric acid shown in the Supplement (Fig. S4). Overall, the results of EQSAM4clim and ISORROPIA II are close to those of Fig. 10.23 of SP2006. Minor differences in ammonium, nitrate and water occur at ammonia concentrations above 6 [µg m$^{-3}$ (air)], since the gaseous uptake of NH$_3$ and HNO$_3$ on saturated solutions is not considered for EQSAM4clim, see Supplement (Sect. S2). The EQSAM4clim results are for ammonia concentrations below 6 [µg m$^{-3}$ (air)] somewhat closer to those of SP2006 (see their Fig. 10.23), while the opposite is true for higher ammonia concentrations.

3.3 Variable solute concentrations (20 cases): ISORROPIA II and EQUISOLV II

To scrutinize the differences between EQSAM4clim and ISORROPIA II, we further evaluate 20 variable mixed solution cases, following the comparison presented in Xu et al. (2009),
using the corresponding sulfate molar ratios of their Table 3. Figures 6 and 7 show the modelling results for the following RHs: 10, 20, 30, 40, 50, 60, 70, 80, 90, 95 [%] in comparison to EQUISOLV II for case 16, which corresponds to domain D1 of Tables 2 and 3. The remaining cases are shown in Figs. S5–S7 in the Supplement. The aerosol composition is calculated from the full gas-liquid-solid equilibrium partitioning with the assumption that the aerosol lies on the deliquescence branch. Again, this comparison indicates that the relatively largest differences in the aerosol water mass calculations occur in the mutual deliquescence humidity range, while the general water uptake above this range is rather similar for all three equilibrium models (Fig. 6). This finding is also supported by the solid and total PM and confirmed by the total aerosol ammonium and nitrate comparison (Fig. 7).

For all cases, the results of EQSAM4clim are close to the results of ISORROPIA II and EQUISOLV II (see also Supplement, Sect. S3.3).

The comparison of total nitrate and aerosol ammonium (Fig. 7) further reveals that also the semi-volatile compounds are rather well represented by EQSAM4clim, despite the underlying simplified analytical approach. Note that the lumped concentrations of the semi-volatile ions are shown only for the most complex cases, i.e., for the 10 sulfate neutral/poor cases (11–20). The common treatment among these EQMs is that both ammonium nitrate and ammonium chloride can be formed only when sulfate has been fully neutralized. A surplus of ammonia must exist to neutralize nitric acid and/or hydrochloric acid. Thus, their neutralization also critically depends on the presence of the non-volatile, mineral cations, i.e., Ca^{2+}, Mg^{2+}, K^{+}, Na^{+}, which have been considered for certain cases (e.g., case 16). Depending on the sulfate loadings, these cations can practically determine the whole gas-liquid-solid-partitioning and the water uptake (Metzger et al., 2006).

3.4 Field observations (MINOS campaign, 184 cases): ISORROPIA II

To scrutinize further mineral rich cases and to extend our model inter-comparison to size-resolved aerosol observations, we further apply both gas/aerosol partitioning schemes to (184) field measurements of the Mediterranean INtensive Oxidant Study (MINOS) that
were obtained during a campaign in Crete in the period of 27 July to 25 August 2001 (Lelieveld et al., 2002; Salisbury et al., 2003). Figures 8 and 9 compare the fine and coarse mode total particulate matter [µmol m\(^{-3}\) (air)], the predicted associated water mass [µg m\(^{-3}\) (air)] and the residual gases [µmol m\(^{-3}\) (air)], i.e., [NH\(_3\)]\(_{g}\), [HNO\(_3\)]\(_{g}\), [HCl]\(_{g}\) obtained for EQSAM4clim from Eqs. (6)–(12) with the results of ISORROPIA II and the MINOS observations, following Metzger et al. (2006). For a general description of the measurements and the modeling set-up we refer to this article Metzger et al. (2006). Here we apply both gas-aerosol partitioning models at the same level of complexity by considering the ammonium-sulfate-nitrate-chloride-sodium-calcium-magnesium-potassium-water system, i.e., F4 and C4 in Table 1 of Metzger et al. (2006). Note that we omit here organic compounds for a consistent model inter-comparison, despite their relevance for the absolute comparison with observations. We refer to Metzger et al. (2006) for the influence of organic compounds on the ammonium partitioning during the MINOS campaign. Overall, also the size-resolved aerosol results of EQSAM4clim and ISORROPIA II are in close agreement with each other and reproduce field observations (see also the Table 5 for the statistics and Appendix C for the evaluation metrics and the additional comparison in the Supplement, Sect. S3.4).

3.5 EMAC vs. satellite and AERONET observations

To extend the model inter-comparison of EQSAM4clim and ISORROPIA II to global modelling applications, we use the atmospheric chemistry-climate model EMAC in a setup following Abdelkader et al. (2015). Both gas-aerosol partitioning schemes are implemented in the GMXe aerosol microphysics submodule, as described in Pringle et al. (2010) – fully coupled with the EMAC chemistry, transport and radiation schemes. EQSAM4clim and ISORROPIA II are embedded in EMAC in exactly the same way, so that a direct comparison of the global modeling results can be made. Deviations can be fully explained by differences in the gas/aerosol partitioning and water uptake calculation approach.

To evaluate the EMAC results, we compare the aerosol optical depth (AOD) to three independent observational data sets, i.e., two satellites products, i.e., MODIS and MISR,
and one ground based product, i.e., from the AErosol RObotic NETwork (AERONET), http://aeronet.gsfc.nasa.gov. The AOD, or extinction coefficient, is a measure of radiation scattering and absorption at different wavelengths and sensitive to the gas-liquid-solid partitioning and aerosol hygroscopic growth. MODIS monitors the ambient AOD over the oceans and over a portion of the continents (see http://modis-atmos.gsfc.nasa.gov/). The MISR aerosol product is available globally. Both data products (and further information) are available from http://disc.sci.gsfc.nasa.gov/giovanni.

Figure 10 compares the model simulations and observations for the year 2005 (annual mean based on 5 hourly values). The upper left panel shows the EMAC results based on ISORROPIA II, the upper right panel shows the results based on EQSAM4clim; the results represent two independent simulations with an identical model set-up and spin-up. The AOD observations of MODIS and MISR are shown in the lower left and right panels, respectively. The qualitative comparison already shows that the differences between the two EMAC simulations and the satellite observations is larger than the differences between the two different EMAC simulations (despite the two distinct different gas-aerosol partitioning schemes). This result is supported by the AERONET observations, which are included in Fig. 10 as squares (with the same AOD color scale). With respect to the observations, EMAC slightly underestimates the AOD, mainly over the open oceans, intense biomass burning and dust outbreaks, including the trans-Atlantic dust transport. Although the global dust belt seems to be captured rather well by the EMAC simulations, the current model set-up somewhat underestimates the AOD for the year 2005. The main reason is that we have limited the water uptake only to major inorganic salt compounds (those considered above in Sect. 3.4) for the sake of a consistent model inter-comparison of the two gas/aerosol partitioning schemes. A more complete set-up that includes the water uptake of, e.g., biomass burning and organic compounds, will be presented separately. Nonetheless, considering the differences between the observations and the uncertainty of the AOD products, also these EMAC model predictions of EQSAM4clim and ISORROPIA II seem reasonable.
4 Conclusions

We have successfully extended the $\nu_i$-parameterizations from single to mixed solutions. The novelty of our single parameter framework is given by the fact that only one coefficient per compound is required to solve the multicomponent gas-liquid-solid partitioning. Our results show that this approach is possible, since we use (i) a consistent set of equations that are all based on the mass fraction solubility $\chi_s$ (Eq. A1) and $\nu_i$ (Sect. 2), and (ii) since we can assume $\nu_i$ to be constant (Sect. 2.1 and A4) for the entire range of water activity, $a_w$ (for the $a_w$ parameterization see Eq. 5a of M2012 and Eq. A3). For semi-volatile compounds, we (iii) parameterize the temperature and humidity dependent equilibrium dissociation constant, $K_{p,AN}(T, RH)$, by substituting required activity coefficients with a new equation that is also only based on $\chi_s$ and $\nu_i$ (Eqs. 1–5). The advantage is that $\nu_i$ can be accurately determined from one single data pair, i.e., the widely used solute’s mass fraction solubility and the corresponding $a_w$ – for the latter we use in this work RHD measurements (Sects. 2.1 and A4). With M2012 we have demonstrated that the $\nu_i$-concept allows to accurately determine the aerosol water mass of binary solutions, $m_{w,j}$, for a given solute concentration $n_{j(aq)}$. With this work we have shown that this is also true for the total aerosol water mass of mixed solutions, $m_{w,mix}$ (Eq. 22), by using $\mu_s$ (Eq. A3). Differences to reference calculations are basically caused by the assumptions made on the mixed solution RHD (Sect. 2.6), i.e., the different assumptions on the mutual deliquescence humidity range. Examples that can be verified with a pocket calculator are presented in the Supplement (Sect. S1) – they support the various box and global modeling results of Sects. 3 and S3.

Appendix A: Single solute solutions

A1 Solute mass fraction, $\chi_s$, and solute molality, $\mu_s$

The relation between solute mass fraction $\chi_s$ and solute molality $\mu_s$ is central in our mixed solution parametrization framework (Sect. 2). Both can be expressed through each other
The solute mass fraction, \( \chi_s \), is defined as the mass \([\text{kg}]\) of solute, \( m_s = n_s \times M_s \), relative to the total mass \([\text{kg}]\) of the solution composed out of the mass of solute \( m_s \) and water, \( m_w = n_w \times M_w \):

\[
\chi_s := \frac{m_s}{m_s + m_w} = \left( \frac{m_w}{m_s} + 1 \right)^{-1} = \left( \frac{n_w \times M_w}{n_s \times M_s} + 1 \right)^{-1} = \left( \frac{1}{M_s \times \mu_s} + 1 \right)^{-1} \quad (A1)
\]

\( n_s \) and \( n_w \) [mol] are the number of moles of solute and solvent (water), \( M_s \) and \( M_w \) [kg mol\(^{-1}\)] are the corresponding molar masses of the solute and water, respectively.

The solute molality is defined as the number of moles of solute per kilogram of water, i.e., \( \mu_s \) [mol(solute) kg\(^{-1}\)(H\(_2\)O)]. It can be expressed in terms of the solute mass fraction by:

\[
\mu_s := \frac{n_s}{1 \text{ kg} \text{ H}_2\text{O}} = \frac{n_s}{m_w} = \frac{n_s}{n_w \times M_w} = \frac{1}{M_s \times (1/\chi_s - 1)} \quad (A2)
\]

\( \mu_s \) measurements, tabulated as a function of water activity \( (a_w) \), are used in atmospheric modeling under the assumption that \( a_w \) equals RH to obtain the single solute (partial) aerosol water mass that is in equilibrium with a given amount of the single solute, \( n_s \), at a given RH from Eq. (22); Sect. 2.7. Under this assumption, \( \mu_s \) is a function of RH, actually \( a_w \), but in any case a function of the available water vapor mass, \( m_w \), which is in equilibrium associated with the solute mass, \( m_s \). This illustrated in Figs. A1 and A2 for several electrolytes used in this work; the solid lines refer to \( \mu_s \) measurements, the dotted lines to a parameterization; see Sect. A2. Since for atmospheric applications, the aerosol associated water mass depends on the available water vapor mass, M2012 have expressed the single solute molality as a function of RH and a solute specific coefficient, \( \nu_i \). The M2012 concept is summarized in the following and has been extended to mixed solutions in Sect. 2.

### A2 Parameterization of \( \mu_s \) and \( \chi_s \)

The representation of water activity (M2012) relates \( a_w \) to the solute molality \( \mu_s \) through a single solute specific constant, \( \nu_i \). This is a major advantage compared to other parameterizations, because the number of unknowns is reduced to one. To extend
the \( \nu_i \)-approach to mixed solutions we use the parameterization of solute molality \( \mu_s \) [mol(solute) kg\(^{-1}\)(H\(_2\)O)]. Inverting Eq. (5a) of M2012 allows to express \( \mu_s \) as a function of RH and \( \nu_i \), with \( a_w := \frac{RH}{K_e} \).

\[
\mu_s = \mu_s^0 \times \left( \frac{1}{\mu_s^0 \times M_w \times \nu_i} \times \left( \frac{K_e}{\text{RH}} - A \right) \right)^{\frac{1}{\nu_i}} - B \tag{A3}
\]

The equivalent expression for \( \chi_s \) \([-\] \) is given by inserting Eq. (A3) in Eq. (A1).

\( \mu_s^0 = 1 \) [mol kg\(^{-1}\)] denotes a reference to match units. \( M_w \) [kg mol\(^{-1}\)] is the molar mass of water and \( \nu_i \) a single solute specific constant. \( K_e \) denotes the Kelvin-term (see Sect. A5) and depends on the mass equivalent hygroscopic growth factor, GF (see Sect. A6). The terms \( A \) and \( B \) are defined by M2012; see their Eqs. (2) and (3) and are slightly revised (further simplified) in the following.

### A3 Parameterization of \( A \) and \( B \)-terms

To break down the thermodynamics as much as possible, we use a simplified representation of the \( A \) and \( B \)-terms compared to M2012. Throughout this work, we use a \( B \)-term that has been empirically determined to be a function of \( \nu_i \) with the constraint that \( A := 1 \). Here, \( B \) is expressed in terms of the solute mass fraction \( \chi_s \) and defined as:

\[
B := \chi_s \left[ \frac{1}{1 + \nu_i + \chi_s} \right] \tag{A4}
\]

To express \( \chi_s \) in Eq. (A4), we use Eq. (A3) for \( \mu_s \) (right term of Eq. A1).

### A4 Relative Humidity of Deliquescence (RHD)

To pre-determine \( \nu_i \) for our mixed solutions framework we use RHD measurements at \( T_o = 298 \) [K]; see Sects. 2.1 and 2.3 in M2012. To solve our mixed solution framework we calculate the temperature dependency from (Wexler and Potukuchi, 1998):

\[
\text{RHD}(T) = \text{RHD}(T_o) \times \exp \left[ T_{\text{coef}} \times \left( \frac{1}{T} - \frac{1}{T_o} \right) \right] \tag{A5}
\]
The RHD measurements at $T_o = 298$ and the corresponding temperature coefficients are taken from (Fountoukis and Nenes, 2007) and listed in Table 1.

To determine $\nu_i$, we solve Eq. (5b) of M2012, which we include here for completeness:

$$\text{RHD} = \frac{K_e}{\left( A + \mu_s^{w} \times M_w \times \nu_i \times \left[ \frac{1}{\mu_s^{w}} \times \frac{1}{M_s (1/w_s - 1)} + B \right]^{\nu_i} \right)} \quad (A6)$$

The RHD [-] describes the point of water activity, $a_w [-]$, where a solution is saturated. Any excess of solute leads to solute precipitation and co-existence of a solid and liquid phase (see Sects. 2.6 and 2.7). At solute saturation, the solute mass fraction (Eq. A1) is measured by the widely used mass fraction solubility $w_s [-]$. Since the saturation molality $\mu_{s, \text{sat}}$ and the mass fraction solubility $w_s$ are related by Eq. (A2), i.e., $\mu_{s, \text{sat}} = \frac{1}{M_s (1/w_s - 1)}$, M2012 express the RHD in terms of $w_s$ and a single solute specific coefficient, $\nu_i$. For a given $w_s$ and RHD data pair, $\nu_i$ can be accurately determined if Eq. (A6) is solved with a root-finding method (e.g., bisection). This procedure has been detailed in M2012 and only requires one data-pair. To pre-determine $\nu_i$ for all salt compounds used in this work, we follow M2012 and use the $w_s$ and RHD measurements at $T_o = 298$ [K]. The pre-determined $\nu_i$ values used are included in Table 1. To solve our mixed solutions framework we assume $\nu_i$ constant and independent of the temperature. Therefore Eq. (A6) is not needed. It could be used during runtime, e.g., within EQSAM4clim (see Sect. B), to determine $\nu_i$ for different $T$. But this is beyond the scope of this work, since the reference models currently available to evaluate our mixed solution parameterization framework are also based on water activity data at $T_o$.

A5 Kelvin-term

The parameterization of solute molality, $\mu_s$ (introduced by M2012), depends on the so-called Kelvin-term, $K_e [-]$ (Eq. A3), which can be expressed in terms of the growth factor, $g_s$:

$$K_e = \exp \left( \frac{4 \times M_w \times \sigma_{\text{sol}}}{R \times T \times \rho_w \times D_w} \right) = \exp \left( \frac{4 \times M_w \times \sigma_{\text{sol}}}{R \times T \times \rho_w \times g_s \times D_e} \right) \quad (A7)$$
\[ \sigma_{\text{sol}} \, [\text{J} \, \text{m}^{-2}] \] denotes the RH dependent surface tension of the solution droplet. \( T \, [\text{K}] \) is the droplet temperature, \( R = 8.314409 \, [\text{J} \, \text{mol}^{-1} \, \text{K}^{-1}] \) the ideal gas constant, \( M_w \, [\text{kg} \, \text{mol}^{-1}] \) the molar mass and \( \rho_w \, [\text{kg} \, \text{m}^{-3}] \) the density of water. \( D_{\text{wet}} \) and \( D_s \, [\text{m}] \) are the ambient and dry droplet diameter, respectively.

### A6 Growth factor

Equation (A7) depends on the RH dependent (mass equivalent) hygroscopic growth factor, \( g_s \, [-] \). We assume a geometric diameter = mass equivalent diameter of a compact spherical droplet and a constant surface tension of pure water droplets, i.e., \( \sigma_{\text{sol}} = 0.0761 \, [\text{J} \, \text{m}^{-2}] \).

We further assume that the droplet temperature is in equilibrium with the ambient air, and we consider the widely used “volume-additivity”. Then, the volume of the solution droplet can be expressed as the sum of volumes of the dry solute and that of the associated pure (aerosol) water contained in the droplet. The ambient diameter, \( D_{\text{wet}} \), of the droplet can therefore be expressed in terms of the solute dry diameter, \( D_s \), and \( g_s \):

\[
g_s := \frac{D_{\text{wet}}}{D_s} = \left( \frac{V_w + V_s}{V_s} \right)^{1/3} = \left( \frac{V_w}{V_s} + 1 \right)^{1/3} = \left( \frac{\rho_s \times m_w}{\rho_w \times m_s} + 1 \right)^{1/3} = \left( \frac{\rho_s \times m_w}{\rho_w \times m_s} + 1 \right)^{1/3}
\]

\( V_w + V_s \, [\text{m}^3] \) is the total volume of the wet droplet with \( V_s = m_s/\rho_s = n_s \, M_s/\rho_s \) and \( V_w = m_w/\rho_w = n_w \, M_w/\rho_w \, [\text{m}^3] \), i.e., the volumes of the initially dry solute and the associated pure water, respectively. \( m_s \) and \( m_w \, [\text{kg}] \) denote the corresponding solute and water masses, \( M_s \) and \( M_w \, [\text{kg} \, \text{mol}^{-1}] \) the molar masses, \( n_s \) and \( n_w \, [\text{mol}] \) the number of moles, and \( \rho_s \) and \( \rho_w \, [\text{kg} \, \text{m}^{-3}] \) the densities, respectively.

### Appendix B: EQSAM4clim

We apply our new mixed solution parameterization framework (Sect. 2) in the EQuilibrium Simplified Aerosol Model V4 for climate simulations. Selected results are shown in...
Sect. 3, which are extended in the Supplement. EQSAM4clim aims at accurate but numerically efficient water uptake calculations that are applicable to high resolution or long-term modeling at climate time scales, i.e., decades to centuries. In contrast to previous EQSAM versions and all other thermodynamic gas-liquid-solid aerosol partitioning models, EQSAM4clim considers a consistent, simplified mixed solution parameterization, which can be solved analytically. Our key equation of solute molality, $\mu_s$, (Eq. A3), and the entire mixed solution phase partitioning (Sect. 2) can be solved analytically, even with a pen and pocket calculator as demonstrated in the Supplement (Sect. 1). EQSAM4clim (as all other EQSAM versions) builds on the fact that for atmospheric applications, $\mu_s$ can be expressed as a function of Relative Humidity (RH); Sect. A2. This was first demonstrated by Metzger et al. (2002a) (based on Metzger et al., 1999; Metzger 2000; PhD Thesis, University of Utrecht, the Netherlands, http://adsabs.harvard.edu/abs/2000PhDT.......328M, Provided by the SAO/NASA Astrophysics Data System).

To solve our mixed solution framework, we express $g_s$ in terms of $\mu_s$ to reduce the number of unknowns to one, i.e., $\nu_i$; assuming the aerosol dry size $D_s$, temperature ($T$) and relative humidity (RH) are prescribed (e.g., given during run-time as model input). Since $g_s [-]$ is defined as the ratio of wet to dry droplet diameter, it can be expressed in terms of the solute molality (by using either Eq. A2 or Eq. A3). Due to its implicit character in $\mu_s$, solving Eq. (A3) requires iterations. One can apply an efficient root finding algorithm, which converges quickly. Treating $K_e$ and the $B$-term as perturbation, it is possible to truncate after the fourth iteration. Higher accuracy will not improve the results much further. For EQSAM4clim, we solve Eq. (A3) for a given aerosol composition, with $T$, RH and $D_s$ as the unknown variables that are given at model input for each model grid box and time step. Note that we use the $K_e$-term for Eq. (A3), so that EQSAM4clim compares to Para1 of M2012 (see their Table 1).

We solve $\mu_s$ for all compounds listed in Table 1 for a given RH from a two-step approach:

- Step one: $K_e = 1$, and $B = 0$ to obtain the initial $\mu_s$ from Eq. (A3).
Step two, repeated three times: \( \mu_s \) from previous iteration is used to calculate \( K_e \) from Eq. (A7), \( \chi_s \) from Eq. (A1), \( B \) from Eq. (A4). Then a new \( \mu_s \) is obtained from Eq. (A3).

Figures A1 and A2 show \( \mu_s \) obtained from Eq. (A3) (applied within EQSAM4clim) as a function of bulk water activity, i.e., \( K_e = 1 \) and \( a_w = \text{RH} \), for major electrolytes (at \( T = 298.15 K \)). All single solute molalities compare well with the tabulated solute molality data (Sect. A1) used in ISORROPIA II (and other EQMs) for the entire \( a_w \)-range: from the water activity at saturation, i.e., RHD, up to \( a_w = 1 \) (pure water). Furthermore, the results of M2012 can be reproduced with Eq. (A3) using the modified \( B \)-term, Eq. (A4) with \( A = 1 \).

Figure A3 compares the GF obtained with Eq. (A8) (marked as NEW) for pure \( \text{NaCl}^{(cr)} \) and \( \text{(NH}_4\text{)}_2\text{SO}_4^{(cr)} \) particles with a dry diameter \( D_s = 0.05 [\mu\text{m}] \) and \( D_s = 1 [\mu\text{m}] \) against those of M2012 (marked as ACP = Para1) and E-AIM for the subsaturated RH regime with \( RH \leq 97 \% \) (upper panels), and the subsequent regime, i.e., \( 97 \% \leq RH \leq 100 \% \) (lower panels). Figure A4 compares the corresponding wet particle diameter, \( D_{\text{wet}} \), as a function of supersaturation (all at \( T = 298.15 K \)); see description of Figs. 3, 4 and 5 of M2012.

Besides significant computational speed-up, another advantage is that our framework minimizes the number of thermodynamic data that are normally required, and reduces the associated uncertainty, typically required, while it enables a larger flexibility with respect to the extension to other compounds, not considered in this evaluation. EQSAM4clim (v09) is limited to the same salt compounds as ISORROPIA II, so that the single solute parameter \( \mu_s \), which is required to solve the single solute molality, \( \mu_s \) (Eq. A3), can be determined from a single reference data-pair (Sect. A4). In addition, most computations are comprehensive and complex. In contrast, the numerical algorithm of EQSAM4clim is simple and easier to verify, since it does not involve any numerical solution or iteration to solve the gas-liquid-solid partitioning for the reasons summarized in Sect. 2.7.

EQSAM4clim has the advantage of being a short fortran 90 code with approximately 850 lines, including comments (or 8 pages), see Fig. S1 in the Supplement for a sneak preview. Figure S2.1 in the Supplement shows the flow chart of processes and operations; the computational algorithm is summarized in the Supplement (Sect. S2). For comparison, the gas-aerosol partitioning routine ISORROPIA II, also used in EMAC counts
roughly 36300 lines (or approx. 360 pages). For comparison, this is about 1/3 of the source code of the EMAC climate model core (ECHAM5.3.02), which has about 120000 lines of f90 code (both including comments). Last but not least, due to its analytical structure the additional computational costs of EQSAM4clim are negligible for our climate applications, which will be detailed and presented separately.
Appendix C: Evaluation metrics

- **RMSE** – Root Mean Square Error between the model \((m)\) and the observations \((o)\):
  \[ RMSE = \sqrt{\frac{1}{N} \sum (X_m - X_o)^2} \]  
  \[(C1)\]

- **\(\sigma\)** – Standard deviation of the model \((m)\) and the observed \((o)\) value:
  \[ \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (X_i - \bar{X})^2}, \quad \text{where} \quad \bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i \]  
  \[(C2)\]

- **\(R\)** – Correlation coefficient between the model \((m)\) and the observations \((o)\):
  \[ R = \frac{\sum_{i=1}^{N} (X_i^m - \bar{X}^m)(X_i^o - \bar{X}^o)}{\sqrt{\sum_{i=1}^{N} (X_i^m - \bar{X}^m)^2 \sum_{i=1}^{N} (X_i^o - \bar{X}^o)^2}} \]  
  \[(C3)\]

- **MBE** – Mean Bias Error between the model \((m)\) and the observations \((o)\):
  \[ MBE = \frac{1}{N} \sum (X_m - X_o) \]  
  \[(C4)\]

- **\(r\)** – Geometric mean of the model \((m)\) and the observations \((o)\):
  \[ r = \sqrt[\sqrt{N}]{\prod_{i=1}^{N} X_i} \]  
  \[(C5)\]

- **GFE** – Growth Factorial Error:
  \[ GFE = \frac{1}{N} \sum \frac{|(X_m - X_o)|}{X_m + X_o} \]  
  \[(C6)\]
- SS1 – Skill score between the model ($m$) and the observations ($o$):

$$SS1 = \frac{4(1 + R)}{(\sigma_f + 1/\sigma_f)^2(1 + R_0)}, \quad \text{where} \quad \sigma_f = \frac{\sigma_o}{\sigma_m} \quad R_0 = 0.0$$ (C7)

PF2 is fraction of the number of points within a factor of two of the observations, PF10 is fraction of the number of points within a factor of ten of the observations, and NPoints is the number of points used.

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References


Table 1. Thermodynamic data—Units see text (Sect. 2.1): Stoichiometric coefficient $\nu_i [-]$, the ion-pair charge $Z_s [-]$, the single solute parameter $\nu_i [-]$, the mass fraction solubility in percent $W_s [%] (w_s = W_s/100)$, the molar masses $M_s [\text{kg mol}^{-1}]$, the densities $D_s [\text{kg m}^{-3}]$, RHD($T_o$) [-] at reference temperature $T_o = 298.15 [\text{K}]$, and the corresponding temperature coefficients, $T_{\text{coef[RHD]}} [-]$. The $\nu_i$ values have been obtained from the RHD and $W_s$ values (at $T_o = 298 [\text{K}]$) following the method described in Metzger et al. (2012). The RHD values are taken from Fountoukis and Nenes (2007); the $W_s$ and all other values are taken from the Handbook of Chemistry and Physics (Lide, 2005).
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<td>186.00</td>
<td>80.00</td>
<td>384.00</td>
<td>852.00</td>
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<table>
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<tr>
<th>Species 11–15</th>
<th>Na₃H(SO₄)₂</th>
<th>Na₂SO₄</th>
<th>NaHSO₄</th>
<th>NaNO₃</th>
<th>NaCl</th>
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</thead>
<tbody>
<tr>
<td>νₛ</td>
<td>Zₛ</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>νᵢ</td>
<td>–</td>
<td>1.278762</td>
<td>1.293906</td>
<td>1.160345</td>
<td>1.160345</td>
</tr>
<tr>
<td>Wₛ</td>
<td>–</td>
<td>21.94</td>
<td>66.18</td>
<td>47.70</td>
<td>26.47</td>
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<tr>
<td>Mₛ</td>
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<td>0.142050</td>
<td>0.120070</td>
<td>0.085000</td>
<td>0.058440</td>
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<tr>
<td>Dₛ</td>
<td>2565</td>
<td>2700</td>
<td>2430</td>
<td>2260</td>
<td>2170</td>
</tr>
<tr>
<td>RHD</td>
<td>–</td>
<td>0.9300</td>
<td>0.5200</td>
<td>0.7379</td>
<td>0.7528</td>
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<tr>
<td>Tₖₜₑₒᶠ</td>
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<td>80.00</td>
<td>-45.00</td>
<td>304.00</td>
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### Table 1. Continued.

<table>
<thead>
<tr>
<th>Species 16–20</th>
<th>K$_3$H(SO$_4$)$_2$</th>
<th>K$_2$SO$_4$</th>
<th>KHSO$_4$</th>
<th>KNO$_3$</th>
<th>KCl</th>
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<tbody>
<tr>
<td>$\nu_s$</td>
<td>$Z_s$</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>–</td>
<td>1.286445</td>
<td>1.308499</td>
<td>1.014102</td>
<td>1.256989</td>
</tr>
<tr>
<td>$W_s$</td>
<td>–</td>
<td>10.71</td>
<td>33.60</td>
<td>27.69</td>
<td>26.23</td>
</tr>
<tr>
<td>$M_s$</td>
<td>0.310444</td>
<td>0.174266</td>
<td>0.136178</td>
<td>0.101108</td>
<td>0.074548</td>
</tr>
<tr>
<td>$D_s$</td>
<td>2490</td>
<td>2660</td>
<td>2320</td>
<td>2110</td>
<td>1988</td>
</tr>
<tr>
<td>RHD</td>
<td>–</td>
<td>0.9750</td>
<td>0.8600</td>
<td>0.9248</td>
<td>0.8426</td>
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<tr>
<td>$T_{\text{coef}}$</td>
<td>–</td>
<td>35.60</td>
<td>0</td>
<td>0</td>
<td>159.00</td>
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<table>
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<tr>
<th>Species 21–25</th>
<th>n/a</th>
<th>CaSO$_4$</th>
<th>n/a</th>
<th>Ca(NO$_3$)$_2$</th>
<th>Ca(Cl)$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_s$</td>
<td>$Z_s$</td>
<td>–</td>
<td>–/–</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>–</td>
<td>1.271828</td>
<td>–</td>
<td>1.586562</td>
<td>2.024869</td>
</tr>
<tr>
<td>$W_s$</td>
<td>–</td>
<td>0.21</td>
<td>–</td>
<td>59.02</td>
<td>44.84</td>
</tr>
<tr>
<td>$M_s$</td>
<td>–</td>
<td>0.136150</td>
<td>–</td>
<td>0.164100</td>
<td>0.110980</td>
</tr>
<tr>
<td>$D_s$</td>
<td>–</td>
<td>2960</td>
<td>–</td>
<td>2500</td>
<td>2150</td>
</tr>
<tr>
<td>RHD</td>
<td>–</td>
<td>0.9900</td>
<td>–</td>
<td>0.4906</td>
<td>0.2830</td>
</tr>
<tr>
<td>$T_{\text{coef}}$</td>
<td>–</td>
<td>0</td>
<td>–</td>
<td>509.40</td>
<td>551.10</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Species 25–30</th>
<th>n/a</th>
<th>MgSO$_4$</th>
<th>n/a</th>
<th>Mg(NO$_3$)$_2$</th>
<th>Mg(Cl)$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_s$</td>
<td>$Z_s$</td>
<td>–</td>
<td>–/–</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\nu_i$</td>
<td>–</td>
<td>1.435281</td>
<td>–</td>
<td>1.878693</td>
<td>2.107772</td>
</tr>
<tr>
<td>$W_s$</td>
<td>–</td>
<td>26.31</td>
<td>–</td>
<td>41.59</td>
<td>35.90</td>
</tr>
<tr>
<td>$M_s$</td>
<td>–</td>
<td>0.120375</td>
<td>–</td>
<td>0.148325</td>
<td>0.095205</td>
</tr>
<tr>
<td>$D_s$</td>
<td>–</td>
<td>2660</td>
<td>–</td>
<td>2300</td>
<td>2325</td>
</tr>
<tr>
<td>RHD</td>
<td>–</td>
<td>0.8613</td>
<td>–</td>
<td>0.5400</td>
<td>0.3284</td>
</tr>
<tr>
<td>$T_{\text{coef}}$</td>
<td>–</td>
<td>–714.45</td>
<td>–</td>
<td>230.20</td>
<td>42.23</td>
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Table 2. Chemical domains (introduced in Sect. 2.2).

<table>
<thead>
<tr>
<th>Domain</th>
<th>tCAT Conditions</th>
<th>tSO₄ Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>D4 SULFURIC ACID ONLY</td>
<td>tCAT &lt; MIN AND tHSO₄ ≥ MIN</td>
<td></td>
</tr>
<tr>
<td>D3 SULFATE VERY RICH</td>
<td>tCAT ≥ MIN AND tCAT &lt; tHSO₄</td>
<td></td>
</tr>
<tr>
<td>D2 SULFATE RICH</td>
<td>tCAT ≥ tHSO₄ AND tCAT &lt; tSO₄</td>
<td></td>
</tr>
<tr>
<td>D1 SULFATE NEUTRAL</td>
<td>tCAT ≥ tSO₄</td>
<td></td>
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</table>
Table 3. Neutralization reaction order for Table 2 (introduced in Sect. 2.3).

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4. Na2SO4</td>
<td>5. (NH4)2SO4</td>
<td>6. Ca(NO3)2</td>
</tr>
<tr>
<td></td>
<td>7. Mg(NO3)2</td>
<td>8. KNO3</td>
<td>9. NaNO3</td>
</tr>
<tr>
<td></td>
<td>10. NH4NO3</td>
<td>11. Ca(Cl)2</td>
<td>12. Mg(Cl)2</td>
</tr>
<tr>
<td></td>
<td>13. KCl</td>
<td>14. NaCl</td>
<td>15. NH4Cl</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7. (NH4)2SO4</td>
<td>8. NH4HSO4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D3</th>
<th>1. CaSO4</th>
<th>2. MgSO4</th>
<th>3. KHSO4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4. NaHSO4</td>
<td>5. NH4HSO4</td>
<td>6. H—HSO4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D4</th>
<th>1. H2SO4</th>
<th></th>
<th></th>
</tr>
</thead>
</table>

D1 = Sulfate Neutral
D2 = Sulfate Rich
D3 = Sulfate Very Rich
D4 = Sulfuric Acid
Table 4. Equilibrium dissociation constants [ppbv$^2$] and T-coefficients [–]; from Fountoukis and Nenes (2007).

<table>
<thead>
<tr>
<th></th>
<th>$K_p^o$</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH$_4$NO$_3$</td>
<td>57.46</td>
<td>-74.38</td>
<td>6.120</td>
</tr>
<tr>
<td>NH$_4$Cl</td>
<td>1.086</td>
<td>-71.00</td>
<td>2.400</td>
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Table 5. MINOS aerosol statistics (see Figs. 8, 9 of Sec. 3.4; Appendix C for the evaluation metrics): EQSAM4clim (EQ4c) and ISORROPIA II (ISO2) versus MINOS observations (Aug 2001).

<table>
<thead>
<tr>
<th></th>
<th>HNO3</th>
<th>NH3</th>
<th>HCl</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aerosol Fine Mode</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EQ4c</td>
<td>ISO2</td>
<td>EQ4c</td>
<td>ISO2</td>
</tr>
<tr>
<td>Mean</td>
<td>19.86 ± 12.89</td>
<td>20.17 ± 13.05</td>
<td>74.16 ± 63.93</td>
<td>74.91 ± 63.41</td>
</tr>
<tr>
<td>Mean</td>
<td>19.56 ± 13.17</td>
<td>19.56 ± 13.17</td>
<td>41.13 ± 40.55</td>
<td>41.13 ± 40.55</td>
</tr>
<tr>
<td>MBE</td>
<td>18.81 ± 0.24</td>
<td>17.01 ± 0.25</td>
<td>37.35 ± 1.48</td>
<td>51.10 ± 0.43</td>
</tr>
<tr>
<td>RMSE</td>
<td>16.15 ± 0.26</td>
<td>16.15 ± 0.26</td>
<td>32.33 ± 0.27</td>
<td>32.33 ± 0.27</td>
</tr>
<tr>
<td>R</td>
<td>0.98</td>
<td>0.99</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>MBE</td>
<td>0.30</td>
<td>0.61</td>
<td>33.07</td>
<td>33.78</td>
</tr>
<tr>
<td>GFE</td>
<td>0.04</td>
<td>0.05</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>SS1</td>
<td>0.99</td>
<td>0.99</td>
<td>0.67</td>
<td>0.68</td>
</tr>
<tr>
<td>PM</td>
<td>1.00</td>
<td>1.00</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td>NPOINTS</td>
<td>124</td>
<td>124</td>
<td>122</td>
<td>122</td>
</tr>
<tr>
<td><strong>Aerosol Coarse Mode</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EQ4c</td>
<td>ISO2</td>
<td>EQ4c</td>
<td>ISO2</td>
</tr>
<tr>
<td>Mean</td>
<td>13.39 ± 13.73</td>
<td>9.55 ± 12.34</td>
<td>43.57 ± 40.39</td>
<td>40.76 ± 41.05</td>
</tr>
<tr>
<td>Mean</td>
<td>19.56 ± 13.17</td>
<td>19.56 ± 13.17</td>
<td>41.13 ± 40.55</td>
<td>41.13 ± 40.55</td>
</tr>
<tr>
<td>MBE</td>
<td>5.79 ± 0.76</td>
<td>0.00 ± 12.54</td>
<td>35.09 ± 0.27</td>
<td>27.97 ± 0.46</td>
</tr>
<tr>
<td>RMSE</td>
<td>16.15 ± 0.26</td>
<td>16.15 ± 0.26</td>
<td>32.33 ± 0.27</td>
<td>32.33 ± 0.27</td>
</tr>
<tr>
<td>R</td>
<td>0.47</td>
<td>0.34</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>MBE</td>
<td>-2.09</td>
<td>-10.01</td>
<td>3.08</td>
<td>0.24</td>
</tr>
<tr>
<td>GFE</td>
<td>0.05</td>
<td>0.59</td>
<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td>SS1</td>
<td>0.73</td>
<td>0.67</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>PM</td>
<td>0.02</td>
<td>0.37</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>NPOINTS</td>
<td>124</td>
<td>124</td>
<td>122</td>
<td>122</td>
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**Table A1.** List of names and abbreviations.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Name</th>
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<tbody>
<tr>
<td>AOD</td>
<td>Aerosol Optical Depth</td>
</tr>
<tr>
<td>AERONET</td>
<td>AErosol RObotic NETwork [<a href="http://aeronet.gsfc.nasa.gov">http://aeronet.gsfc.nasa.gov</a>]</td>
</tr>
<tr>
<td>CNN</td>
<td>Cloud Condensation Nuclei</td>
</tr>
<tr>
<td>CPU</td>
<td>Computational Performance Unit</td>
</tr>
<tr>
<td>EQMs</td>
<td>Thermodynamic equilibrium models</td>
</tr>
<tr>
<td>E-AIM</td>
<td>(Wexler and Clegg, 2002), [<a href="http://www.aim.env.uea.ac.uk/aim/aim.php">http://www.aim.env.uea.ac.uk/aim/aim.php</a>]</td>
</tr>
<tr>
<td>EQSAM</td>
<td>Equilibrium Simplified Aerosol Model (Metzger et al., 2002b)</td>
</tr>
<tr>
<td>EQSAM4clim</td>
<td>Equilibrium Simplified Aerosol Model (Version 4) for Climate Simulations (this work)</td>
</tr>
<tr>
<td>EQUISOLV II</td>
<td>Jacobson (1999)</td>
</tr>
<tr>
<td>EMAC</td>
<td>Atmospheric Chemistry-climate model</td>
</tr>
<tr>
<td>GMXe</td>
<td>Global Modal-aerosol eXtension [Pringle et al., 2010]</td>
</tr>
<tr>
<td>ISORROPIA II</td>
<td>Fountoukis and Nenes (2007)</td>
</tr>
<tr>
<td>MODIS</td>
<td>satellite data [<a href="http://modis-atmos.gsfc.nasa.gov/">http://modis-atmos.gsfc.nasa.gov/</a>]</td>
</tr>
<tr>
<td>MISR</td>
<td>global satellite data [<a href="http://disc.sci.gsfc.nasa.gov/giovanni">http://disc.sci.gsfc.nasa.gov/giovanni</a>]</td>
</tr>
<tr>
<td>MINOS</td>
<td>Mediterranean INTensive Oxidant Study [Lelieveld et al., 2002; Salisbury et al., 2003]</td>
</tr>
<tr>
<td>NRO</td>
<td>neutralization reaction order (Sect. 2.3)</td>
</tr>
<tr>
<td>M2012</td>
<td>Metzger et al. (2012)</td>
</tr>
<tr>
<td>SP2006</td>
<td>Seinfeld and Pandis (2006)</td>
</tr>
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Table A2. List of greek symbols.

<table>
<thead>
<tr>
<th>Greek Symbol</th>
<th>Name</th>
<th>Unit</th>
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<tbody>
<tr>
<td>$\nu_s$</td>
<td>solute specific constant (introduced by M2012)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>stoichiometric coefficient of solute ($\pm$ion-pair)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>molality of solute</td>
<td>[mol kg$^{-1}$(H$_2$O)]</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>reference molality of 1 mole of solute (considering stoichiometry)</td>
<td>[mol kg$^{-1}$(H$_2$O)]</td>
</tr>
<tr>
<td>$\mu_{sat}$</td>
<td>saturation molality of solute</td>
<td>[mol kg$^{-1}$(H$_2$O)]</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>density of solute</td>
<td>[kg m$^{-3}$]</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>density of water</td>
<td>[kg m$^{-3}$]</td>
</tr>
<tr>
<td>$\sigma_{sol}$</td>
<td>surface tension of the solution droplet</td>
<td>[J m$^{-2}$]</td>
</tr>
<tr>
<td>$\chi_s$</td>
<td>solute mass fraction, referring to the solute’s dry mass</td>
<td>[-]</td>
</tr>
<tr>
<td>$\chi_{sat}$</td>
<td>solute mass fraction, referring to the solute’s dry mass at saturation</td>
<td>[-]</td>
</tr>
<tr>
<td>$^{sat}$</td>
<td>superscript, indicator for saturation</td>
<td></td>
</tr>
<tr>
<td>$(cr)$</td>
<td>subscript, phase indicator for anhydrous (solid = crystalline = cr)</td>
<td></td>
</tr>
<tr>
<td>$(aq)$</td>
<td>subscript, phase indicator for aqueous phase</td>
<td></td>
</tr>
<tr>
<td>(g)</td>
<td>subscript, phase indicator for gas phase</td>
<td></td>
</tr>
</tbody>
</table>
### Table A3. List of symbols.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$A$-term of Eq. (A4), introduced with Sect. A3</td>
<td>[-]</td>
</tr>
<tr>
<td>$B$</td>
<td>$B$-term of Eq. (A4), introduced with Sect. A3</td>
<td>[-]</td>
</tr>
<tr>
<td>$a_w$</td>
<td>water activity (Raoult-term)</td>
<td>[-]</td>
</tr>
<tr>
<td>$D_s$</td>
<td>dry droplet diameter of the solute</td>
<td>[m]</td>
</tr>
<tr>
<td>$D_{wet}$</td>
<td>wet droplet diameter of the solution</td>
<td>[m]</td>
</tr>
<tr>
<td>$K_e$</td>
<td>surface or Kelvin-term of the solution</td>
<td>[-]</td>
</tr>
<tr>
<td>$m_s$</td>
<td>crystalline mass of solute</td>
<td>[kg]</td>
</tr>
<tr>
<td>$m_w$</td>
<td>aqueous mass of water (solvent)</td>
<td>[kg]</td>
</tr>
<tr>
<td>$M_s$</td>
<td>molar mass of solute</td>
<td>[kg mol$^{-1}$]</td>
</tr>
<tr>
<td>$M_w$</td>
<td>molar mass of water</td>
<td>[kg mol$^{-1}$]</td>
</tr>
<tr>
<td>$n_s$</td>
<td>moles of solute</td>
<td>[mol]</td>
</tr>
<tr>
<td>$\sum_i n_i$</td>
<td>summation over all moles of solutes</td>
<td>[mol]</td>
</tr>
<tr>
<td>$n_w$</td>
<td>moles of water</td>
<td>[mol]</td>
</tr>
<tr>
<td>GF</td>
<td>Growth Factor</td>
<td>[-]</td>
</tr>
<tr>
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<td>[%]</td>
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<tr>
<td>RH</td>
<td>fractional relative humidity (as used in equations)</td>
<td>[-]</td>
</tr>
<tr>
<td>RHD</td>
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<tr>
<td>$S$</td>
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<td>critical supersaturation in percent</td>
<td>[%]</td>
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<td>reference temperature in Kelvin</td>
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<td>$T$</td>
<td>temperature in Kelvin</td>
<td>[K]</td>
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<tr>
<td>$w_s$</td>
<td>mass fraction solubility, referring to the solute’s dry mass required for saturation</td>
<td>[-]</td>
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<tr>
<td>$W_s$</td>
<td>mass fraction solubility, referring to the solute’s dry mass required for saturation</td>
<td>[%]</td>
</tr>
<tr>
<td>tCAT</td>
<td>total cations (for chemical domains; Sect. 2.2)</td>
<td>[mol m$^{-3}$(air)]</td>
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Figure 1. NH$_4$NO$_3$ equilibrium dissociation constant as function of $T$ at RH < RH(AN). Red crosses show the values of EQSAM4clim, green squares refer to ISORROPIA II and the blue circles show Eq. (10.91) of Seinfeld and Pandis (2006); see their Fig. 10.19, respectively.
Figure 2. NH$_4$NO$_3$ equilibrium dissociation constant as function of RH (at $T = 298.15$ [K]) for various ionic strength factors ($Y$) defined in Seinfeld and Pandis (2006); see their Fig. 10.21, respectively. Red crosses show the results of EQSAM4clim, green squares those of ISORROPIA II. The line-points refer to pure ammonium nitrate ($Y = 1$). The vertical line at RH = 65 [%] indicates the solid-liquid phase partitioning threshold for the mixed solution, shown in Seinfeld and Pandis (2006). The ordinate values refer to the product of $[\text{NH}_3]_{(\text{g,AN})} \times [\text{HNO}_3]_{(\text{g,AN})}$ that are obtained at end of the gas-liquid-solid NH$_4$NO$_3$-partitioning of ISORROPIA II, and by Eq. (9) for EQSAM4clim.
Figure 3. Results of EQSAM4clim (red crosses) and ISORROPIA II (green squares) for two idealized gas-liquid-solid partitioning examples: Single solute (binary) solution of pure NH₄NO₃ (upper panels) and mixed solution of NH₄NO₃ and (NH₄)₂SO₄ with the concentration of each compound fixed to 1 [µmol m⁻³(air)] at T = 298.15 K. The left panels show the NH₄NO₃ concentration in [µmol m⁻³(air)], the right panels show the corresponding particulate mass [µg m⁻³(air)]. The mixed solution RHD described in Sect. 2.6 has been neglected for EQSAM4clim for this case, since this figure presents only an example for Sects. 2.4 and 2.5 (Sect. 2.6 is considered below).
Figure 4. Results of EQSAM4clim (red crosses) and ISORROPIA II (green squares) for the total aerosol water mass $m_{w,mix}$ [kg m$^{-3}$ (air)] obtained by Eq. (22) for the mixed solution of NH$_4$NO$_3$ and (NH$_4$)$_2$SO$_4$ shown in Fig. 3 (lower panels). The results are shown for RH = 50–97 [%] (large panels) and for the RH = 95–99.5 [%] (small panel). The results of E-AIM (web version) (blue circles) are included for comparison. The mixed solution RHD has been obtained for EQSAM4clim from Eqs. (13)–(22) and are based on measured MDRH values for ISORROPIA II. The mutual deliquescence range of EQSAM4clim and ISORROPIA II (described in Sect. 2.6) differs from those of E-AIM (web version: http://www.aim.env.uea.ac.uk/aim/aim.php. This figure is extended by Fig. S3.
Figure 5. Mixed solution composition of NH₄NO₃ and (NH₄)₂SO₄ as a function of total ammonia at \( T = 298.15 \, [K] \) and \( RH = 70 \, [%] \), as defined in Seinfeld and Pandis (2006) for their Fig. 10.23. EqsAM4clim (red crosses) and ISORROPIA II (green squares) for \([TS] = [TN] = 10 \, [\mu g \, m^{-3} \, (air)]\). Note that at zero ammonia, H₂SO₄ is extended by at a maximum; shown in Fig. S4.
Figure 6. Comparison – case 16. Bulk aerosol water mass as a function of RH for various different sulfate molar ratios, fixed for the entire RH range (at constant $T = 298.15\,\text{K}$). The dry concentration ratios of sulfate are, $\frac{\text{NH}_4}{\text{HSO}_4} = 2.0$, $\frac{\text{NO}_3}{\text{HSO}_4} = 1.0$, $\frac{\text{NaCl}}{\text{HSO}_4} = 0.5$, $\frac{\text{K}}{\text{HSO}_4} = 0.04$, $\frac{\text{Ca}}{\text{HSO}_4} = 0.02$, $\frac{\text{Mg}}{\text{HSO}_4} = 0.01$ and corresponds to domain 1 of Table 2. This figure is extended to various (20 cases) sulfate molar ratios that are shown in Fig. S5 and correspond to Table 3 of Xu et al. (2009).
Figure 7. **EQUISOLV II** Comparison – case 16. Bulk aerosol nitrate \((\text{ul})\), ammonium \((\text{ur})\), total solid PM \((\text{II})\), liquid + solid PM \((\text{Ir})\) as a function of RH for the sulfate molar ratios shown in Fig. 6. This figure is extended by Figs. S6 and S7.
Figure 8. Observed and simulated total particulate matter [$\mu$mol m$^{-3}$(air)] and the predicted associated water mass [$\mu$g m$^{-3}$(air)] for the observed aerosol fine and coarse modes; EQSAM4clim (EQ4c), ISORROPIA II (ISO2), MINOS observations (black circles). This figure is extended to various other aerosol properties by Figs. S8 and S9 in the Supplement.
Figure 9. Residual gases $[\text{nmol m}^{-3}(\text{air})]$ corresponding to Fig. 8.
**Figure 10.** EMAC AOD vs. MODIS, MISR and AERONET (550 nm, annual mean 2005).
Figure A1. Single solute molality as a function of water activity for several electrolytes: (NH₄)HSO₄, NaHSO₄, NH₄NO₃, NaNO₃ (at \( T = 298.15 \text{K} \)) calculated with EQSAM4clim from Eqs. (A3) and (A6) compared to tabulated molality and RHD measurements of ISORROPIA II used to determine \( \nu_i \).
**Figure A2.** Fig. A1 continued for \((\text{NH}_4)_2\text{SO}_4, \text{Na}_2\text{SO}_4, \text{NH}_4\text{Cl}, \text{NaCl}\) (at \(T = 298.15\text{ K}\)).
Figure A3. Growth factor of pure NaCl (cr) and (NH₄)₂SO₄ (cr) particles with a dry diameter $D_s = 0.05 \mu m$ (UL) and $D_s = 1 \mu m$ (UR) for RH ≤ 97 [%]. Lower panels show the corresponding values within the subsaturated regime, i.e. 97 ≤ RH ≤ 100 [%]. The results of the parameterization used by EQSAM4clim (labeled NEW) are compared to our ACP 2012 water activity, $a_w$-parameterization (labeled ACP, i.e., Para1 of Table 1 in M2012). The comparison includes the results of E-AIM and extends the corresponding figures of M2012.
Figure A4. Wet particle diameter, $D_{\text{wet}}$, as a function of supersaturation for pure NaCl and $(\text{NH}_4)_2\text{SO}_4$ aerosols with different dry diameters, i.e. $D_s = 0.05$ and $D_s = 1 \mu m$ as shown in Fig. 5 of M2012. $S$ is defined as $S = (s - 1) \times 100 \ [%]$; $s$ is the saturation = fractional RH.
S1 Examples – Semi-volatile compounds

The following three sub-sections provide detailed examples for semi-volatile compounds, described in Sec. 2.4 (main text). The analytical solution is part of our new single parameter gas-liquid-solid partitioning framework and applied in Sec. 3 to mixed aerosol salt solutions through EQSAM4clim (see Appendix B and Sec. S2).

S1.1 Pure NH₄NO₃–gas-solid equilibrium (RH < RHD)

Gas-solid equilibrium of pure ammonium nitrate; NH₄NO₃ (index AN) with RH = 50 % and below the RHDₐₙₙ= 61.83 [%] (Table 1), as illustrated in Fig. 3. The partial pressure product of gaseous ammonia, NH₃(g), and nitric acid, HNO₃(g) must equal or exceed Kp(T) to allow the formation of solid ammonium nitrate, NH₄NO₃(s). This is described by (R1) and the system can be solved with Eqs. (6–12). Note that in this case, only the gas phase concentrations are required to solve (R1), since the concentrations of solids are treated as unity. Here, all concentrations denoted by [ ] are given in [µmol/m³(air)]; the EQSAM4clim computations (see Sec. S2) are performed in [mol/m³(air)]. For comparison with SP2006, we use ppbv. Units, e.g., in [µmol/m³(air)] can be converted to ppbv by multiplication with the molar volume 24.465 [L/mol], and from [µg/m³(air)] by additional division with the compounds molar mass, M₄.

At T₀ = 298.15 K, P₀ = 1 atm and RH = 50 % (i.e., RH < RHDₐₙₙ), with M₄, in units of [g/mol] (and 1 ppbv = 1 nano Liter gas per 1 Liter air = 1 micro Liter(gas) per m³(air)), the unit conversion to [ppbv] from concentrations in [µg(gas)/m³(air)] yields, e.g., for NH₃(g)=17.04 and HNO₃(g)=63.02 (see example on p474 of SP2006):

- NH₃(g) :17.04 [µg(gas)/m³(air)]/17.04 [g/mol]:24.465 [L/mol] = 24.465 [ppbv]
- HNO₃(g):63.02 [µg(gas)/m³(air)]/63.02 [g/mol]·24.465 [L/mol] = 24.465 [ppbv]

The partial pressure product is 24.465²≈600 [ppbv²] and at this T well above the equilibrium value of Kp,AN(T) = 57.5 [ppbv²] (Table 4; Fig. 1), but lower than the value of 655 [ppbv²] given in SP2006, which corresponds to T = 308 [K].

Following SP2006 for this dry case, the amount x [ppbv] of the gaseous concentration Cᵢ [ppbv], which could form a solid, can be directly computed by solving the equation:

\[
\prod_{i=1}^{n} (C_i - x) = K_p(T) \quad (S1)
\]

For ammonium nitrate, (R1) yields a quadratic equation (with \(a \cdot x^2 - 2 \cdot b \cdot x + c = 0\))

\[(24.5 - x)(24.5 - x) = 57.5 \text{ [ppbv}^2\text{]}, \text{i.e.,} \ x^2 - 49x + 543 = 0. \text{ And, upon solving (with} \ x_{1,2} = 0.5 \cdot (-b \pm \sqrt{(b^2 - 4 \cdot a \cdot c)})/a), \text{i.e.,:} \ x_1 = 0.5 \cdot (49 - \sqrt{(2401 - 4 \cdot 543)}) \approx 16.9 \text{[ppbv].} \]

Note that the second solution \(x_2 \approx 32.1\) is to be discarded, since its value is larger than the actual \(C_i\) concentrations. Thus, an equal amount of 16.9 [ppbv] of each NH₃(g) and HNO₃(g) and solid NH₄NO₃(s) would be in equilibrium at this temperature and RH.

In terms of mass loadings, this corresponds to 11.8 and 43.5 and 55.2 [µg/m³(air)], respectively. The mass loadings of the corresponding residual (free) acids are: NH₃(g) = \((24.5 - 16.9)/24.5 \cdot 17 = 5.3\) and HNO₃(g) = \((24.5 - 16.9)/24.5 \cdot 63 = 19.5\) [µg/m³(air)], respectively. This case is illustrated in Fig. 3 (see upper panels); Sec. 2.4.1 and 2.5.
S1.2 Pure NH$_4$NO$_3$-gas-liquid equilibrium (RH ≥ RHD)

Same as the first example, but with RH = 80 % and above the RHD$_{AN}$. In line with
the dry case (Sec. 2.4.1 and S1.1), the amount $x$ [ppbv] of the gaseous concentration
$C_i$ [ppbv], which could form aqueous ammonium nitrate, can be directly computed with
Eqs. (6–12), if Eqs. (2–3) are used to solve $K_p(T, RH)$ (see Sec. 2.4.2). Then, (R-1) can
be analogously solved by using Eq. (S1) of the above dry case (Sec. S1.1):

$$\prod_{i=1}^{n} (C_i - x) = K_p(T, RH)$$

(S2)

For instance, considering the gas-aqueous phase equilibrium of ammonium nitrate at
$RH = 80 \%$, by assuming that the water activity $a_w$ equals RH/100, we can obtain a
value of 10 $[mol_{AN}/kg(H_2O)]$ of the corresponding solute molality, $\mu_{AN}(a_w = 0.8)$, from
measurements, or from Eq. (A3) of Sec. A2; both are shown in Fig. A1 (in Sec. B). Using
Eq. (S2), with $M_s = 0.08$ [kg/mol] (of Table 1), we find for the solute mass fraction
(using Eq. A1) the corresponding value of $\chi_{AN}(a_w = 0.8) = \frac{1}{(0.08\cdot10)+1} = 0.44$. And
from Eq. (3) we obtain, COEF$_{AN}(RH = 80 \%) = 2$·0.44$^2 = 0.39$, so that we can directly
obtain the required value for the T– and RH–dependent ammonium nitrate equilibrium
dissociation constant from Eq. (S2). At $T = 298$ (using the value given in Table 4) we
accordingly find $K_{p,AN}(T = 298, RH = 80 \%) = 57.5$·0.39 ≈ 22 [ppbv$^2$].

Mozurkewich (1993) gives a $K_p(T_o)$ value of 42 [ppbv$^2$], for which we would obtain at
RH=80%, 41.9·0.39 ≈ 16 [ppbv$^2$] and accordingly ≈ 12 [ppbv$^2$] for the value 29.9 [ppbv$^2$],
which was originally given by Pilinis and Seinfeld (1987) (e.g., for use in the thermo-
dynamic model SEQUILIB). For EQSAM4clim, either $K_p(T_o)$ value can be used. The
corresponding value of SP2006 (given in their example below Eq. (10.99)) is ≈ 15 [ppbv$^2$].

Thus, knowing $K_{p,AN}(T, RH)$ one can directly solve the quadratic equation Eq. (S2)
for the aqueous phase analogously to Eq. (S1) for the dry case, without any iterations.
Using $K_{p,AN}(T = 298, RH = 80 \%) ≈ 22$ [ppbv$^2$] and the gas concentration for the solid
example given for Eq. (S1), we obtain for the equilibrium concentrations at RH=80%:

$$(4.25 - x)(4.25 - x) = 5.75$·0.39 ≈ 22, i.e., $x^2 - 49x + 578 = 0$. And, upon solving the
quadratic equation $x_1 = 0.5 \cdot (49 - \sqrt{(2401 - 4 \cdot 578)}) = 19.8$ [ppbv] (the second solution
has to be discarded, since its value is always larger than the actual $C_i$ concentrations).

Using $K_{p,AN}(T = 298, RH = 80 \%) ≈ 16$ [ppbv$^2$] yields $20.5$ [ppbv], while using the ppb
values of the example of SP2006 gives $(5 - x)(6 - x) = 15$, i.e., $x^2 - 11x + 15 = 0$, which
yields $x_1 = 0.5 \cdot (11 - \sqrt{(121 - 4 \cdot 15)}) = 1.6$ [ppbv]. Thus, an equal amount $19.8$ (20.5, 1.6)
[ppbv] of each NH$_3$(g) and HNO$_3$(g) and aqueous NH$_4$NO$_3$(aq) would be in equilibrium at
this temperature and RH. The corresponding mass loadings of the residual (free) acids
are: $24.5 - 19.8)/24.5 · 17 = 3.3$ and $4.25 · 63 = 12.1$ [$\mu g/m^3$ (air)], respectively,
and those of the aqueous cation and anion are: $NH_4^+(aq) = 19.8/24.5 · 18 = 14.6,$ and
$NO_3^-(aq) = 19.8/24.5 · 62 = 50.1$ [$\mu g/m^3$ (air)]. The sum, $14.6 + 50.1 = 64.7$ [$\mu g/m^3$ (air)]
forms NH$_4$NO$_3$(aq) and would be in equilibrium with NH$_3$(g)=17.04 and HNO$_3$(g)=63.02
[$\mu g/m^3$ (air)] at $T = 298.15$ [K] and $RH = 80 \%$, while respectively 3.3 and 12.1
[$\mu g/m^3$ (air)] of NH$_3$(g) and HNO$_3$(g) remain in the gas phase.

Compared to the solid case, the additional formation of 64.7–55.2 ≈ 9.5 [$\mu g/m^3$ (air)]
NH$_4$NO$_3$ corresponds to a change in RH from 50 to 80 [\%]. Fig. 10.21 of SP2006 de-
picts the situation of the RH-dependent equilibrium partitioning. For comparison Fig. 2
(see line-points) and Fig. 3 (left part of the upper panels) reveals the situation for our
EQSAM4clim and the ISORROPIA II applications.
S1.3 EQSAM4clim algorithm: NH$_4$NO$_3$–equilibrium

To provide a complete example for mixed solution cases with an analytical solution of EQSAM4clim (Sec. 2.5), we consider Fig. 3 (Sec. S1.1–S1.2) in terms of the notation of SP2006. For EQSAM4clim, we obtain the NH$_4$NO$_3$(nro) equilibrium concentration by solving Eqs. (6–12), based on chemical domains (Table 2, Sec. 2.2) and the neutralization reaction order (NRO, Table 3, Sec. 2.3).

I. Single Solution, Dry/Wet Case (Fig. 3, upper panels)

For the single solution case shown in Fig. 3, we assume for the EQSAM4clim computations a total (gas+aerosol) cation and anion input concentration of 1 [µmol/m$^3$(air)], i.e., total ammonium $[TA] = [NH_4^+]_{(nro,free)} = 1$ and total nitrate $[TN] = [NO_3^-]_{(nro,free)} = 1$. For this sulfate neutral case (all other ions are zero), we apply domain D1. Solving the NRO for D1 automatically yields only a single reaction, i.e., for NH$_4$NO$_3$, since all other cation and anion products are zero, so that just one ion-pair combination can exist. Considering the ion charge, $[ZN] = [ZA] = 1$, we can obtain from the product $[TA] \cdot [ZA]$ and $[TN] \cdot [ZN]$ the corresponding maximum ammonium nitrate concentration $[NH_4NO_3(nro,max)] := MIN([TA] \cdot [ZA], [TN] \cdot [ZN]) = 1$ [µmol/m$^3$(air)], which is possible for the input concentration with $K_p(T,RH,Y) := 1$ (Sec. 2.5), $T$ and $RH$. The temperature is fixed to $T = 298.15$ K, the RH varies from RH = 30 to RH = 100 [%]. After solving the NRO, $[TA]$ and $[TN]$ are zero over the entire $RH$–range, since here the input concentrations are fixed to 1 [µmol/m$^3$(air)] for this pure NH$_4$NO$_3$ case (binary solution, upper left panel of Fig. 3). To solve a dry and wet case, we again consider $RH = 50$ [%] and $RH = 80$ [%]. According to Table 1, the RHD = 61.83 [%] for NH$_4$NO$_3$ so that at $RH = 50$ [%] only the gas-solid equilibrium partitioning needs to be considered, i.e., (R1), while at $RH = 80$ [%] the gas-liquid equilibrium partitioning, i.e., (R2), is relevant. In EQSAM4clim we solve the equilibrium in molal scale. At $T = 298.15$ K and $RH = 50$ [%], conversion of $K_p(T) = 57.46$ [ppbv$^2$] (Table 4) to the molal scale yields (with $K_p(\RT^2)$, i.e., $57.46$ [ppbv$^2$] *10$^{-18}$/[8.314409/101325] [m$^3$(air)/mol/K] * 298.15 [K]$^2 = 57.46$ *1.67$^{-15}$= 9.6 *10$^{-14}$ [mol/m$^3$(air)]$^2$. Solving Eqs. (6–12), we then obtain for the evaporative loss $[x]$ [µmol/m$^3$(air)] of $[NH_4NO_3(nro,free)]$:

\[ x = 0.5 - \sqrt{(4 \cdot 9.6 \cdot 10^{-14})} = 0.31 \text{ [µmol/m}^3\text{(air)}], \text{with } [TA] = [TN] = 0. \text{ Thus, at } \] 

\[ RH = 50 \text{ [%] }, \text{NH}_4\text{NO}_3(nro) = 1 - 0.31 \approx 0.69 \text{ [µmol/m}^3\text{(air)}], \text{ or 0.69[µmol/m}^3\text{(air)]} \] 

\[ \cdot 80 \text{ [g/mol] } \approx 55.2 \text{ [µg/m}^3\text{(air)] for the total particulate matter (PM). For the aqueous phase at } RH = 80 \text{ [%], we analogously obtain NH}_4\text{NO}_3(naq,nro). \text{ According to the above example (Sec. S1.2), } K_{p,AN} (T = 298, RH = 80\%) = 57.5-0.39 \approx 22 \text{ [ppbv$^2$], so that we get; } \] 

\[ = 22 \cdot 1.67^{-15} = 3.7 \cdot 10^{-14}[\text{mol/m}^3\text{(air)}]^2 \] 

\[ x = 0.5 - \sqrt{(4 \cdot 3.7 \cdot 10^{-14})} \approx 0.19 \text{ [µmol/m}^3\text{(air)]}. \text{ This yields NH}_4\text{NO}_3(naq,nro) \approx 0.81 \text{ [µmol/m}^3\text{(air)]}, \text{ or for the total PM } \approx 0.81 \text{ [µmol/m}^3\text{(air)]} \cdot 80 \text{ [g/mol]} \approx 64.6 \mu g/m^3(air). \text{ These values refer} \] 

\[ \text{respectively to the (upper) left and right panels of Fig. 3, and are close to the values of ISORROPIA II, which are independently computed with a different approach.} \]

II. Mixed Solution, Dry/Wet Case (Fig. 3, lower panels)

For the mixed solution case shown in Fig. 3, we assume for the EQSAM4clim computations a total (gas+aerosol) cation and anion input concentration ([µmol/m$^3$(air)])
of total ammonium $[TA] = [NH_4^+]_{(nro,free)} = 3$, total nitrate $[TN] = [NO_3^-]_{(nro,free)} = 1$, and total sulfate $[TS] = [SO_4^{2-}]_{(nro,free)} = 1$. This sulfate case also falls into domain D1, since other ions are zero. Solving the NRO for D1 yields two reactions. Considering the ion charge, $[ZN] = [ZA] = 1$ and $[ZS] = 2$, we can directly obtain from the NRO and the products $[TA] \cdot [ZA]$, $[TN] \cdot [ZN]$ and $[TS] \cdot [ZS]$ the corresponding maximum concentrations of ammonium sulfate and ammonium nitrate, which is possible for the input concentration with $K_p(T, RH, Y) := 1$ (Sec. 2.5 and S1.3-I):

1. $[TA] \cdot [ZA] = 3$ and $[TS] \cdot [ZS] = 2$:
   $[(NH_4)_2SO_4(nro,max)] := \text{MIN}(3, 2) = 1$

2. $[TA = 3 - 2 = 1] \cdot [ZA = 1] = 1$ and $[TN = 1] \cdot [ZN = 1] = 1$:
   $[NH_4NO_3(nro,max)] := \text{MIN}(1, 1) = 1$

Extending our above example calculation (Sec. S1.3-I) to the mixture of $NH_4NO_3$ of $1 \mu mol/m^3(air)$ of each $NH_4NO_3$ and $(NH_4)_2SO_4$ (lower left panel of Fig. 3), we compute the dry ($RH = 50 \%$) and wet ($RH = 80 \%$) case. For $NH_4NO_3$, the RHD $= 61.83 \%$ and for $(NH_4)_2SO_4$ the RHD $= 79.97 \%$ (Table 1), so that at $RH = 50 \%$ again only the gas-solid equilibrium partitioning needs to be considered. Note that mixed solution effects described in Sec. 2.6 are not considered for the RHD in this example, but they are considered for the gas-liquid and liquid-solid equilibrium partitioning and associated aerosol water uptake examples presented in Sec. 3.

For the mixed solution case, the computation is similar to that discussed above (Sec. S1.3-I), only the ionic strength factor needs to be included for the wet case. For the dry case, the solid equilibrium concentration, $NH_4NO_3(s,nro)$, is identical, since $Y = 1$ (the ionic strength correction factor is defined only for solutions). The total dry aerosol mass therefore is the sum of the $NH_4NO_3(s,nro)$ (from above) and $(NH_4)_2SO_4(s,nro)$ masses, i.e., $PM \approx 55.2 + 132.2 = 187.4 \mu g/m^3(air)$. For the aqueous case, we obtain $Y$ from Eq. (4) (as used for Eq. 5), i.e., we obtain for $1 \mu mol/m^3(air)$ of each, $NH_4NO_3_{(aq,max,nro)}$ and $(NH_4)_2SO_4_{(aq,eq)}$, $Y = \frac{1}{1+3.4} = 0.25$. Using further the above (Sec. S1.2) value for $\mu_{AN}(a_w = 0.8) = 10 \mu mol/kg(H_2O)$ (Fig. A1), we compute the solute mass fraction again from Eq. (A1) as $\chi_{AN}(a_w = 0.8) = (\frac{1}{0.08} - 1)^{-1} = 0.44$. Then, Eqs. (3) and (5) yield $COEF(RH, Y) = 2 \cdot 0.44^2 \approx 0.39$ and $Y^{0.8} = 0.25^{0.8} = 0.33$, so that $K_{p,AN}(T = 298, RH = 80\%) = 57.5 \cdot 0.39 \cdot 0.33 = 7.4 \text{ [ppbv]}^2$, or $7.4 \cdot 167^{-15} = 1.24 \cdot 10^{-14} \text{ [mol/m}^3(air)]$. Solving Eqs. (6–12), we then again obtain for the evaporative loss $x \mu mol/m^3(air)$ of $[NH_4NO_3_{(nro, max)}$):

$$x = 0.5 \cdot \sqrt{4 \cdot 1.24 \cdot 10^{-14}} \approx 0.11 \mu mol/m^3(air)], \text{ with } [TA] = [TN] = 0. \text{ Thus, at } RH = 80 \%, NH_4NO_3_{(aq,nro)} = 1 - 0.11 \approx 0.89 \mu mol/m^3(air), \text{ or for the total (dissolved) } PM \approx 0.89 \mu mol/m^3(air) \cdot 80 g/mol + 132.2 \mu g/m^3(air) = 203.4 \mu g/m^3(air)]$$. These values refer to the (lower) left and right panels of Fig. 3, respectively, and are also close to the results of ISORROPIA II, despite the distinctly different approaches.

S2 Computational algorithm of EQSAM4clim

The EQSAM4clim computational algorithm is summarized as follows (see Fig. S2.2-S1):

- EQSAM4clim (v09), described in Sec. B, considers the salt compounds listed in Table 1. To calculate the gas-liquid-solid partitioning, a pre-calculated (constant) $\nu_t$
cient is used for each compound (ion-pair), which is obtained from a small set of thermodynamic data: stoichiometric coefficient $\nu_s [-]$, the ion-pair charge $Z_s [-]$, the single solute parameter $\nu_i [-]$, the mass fraction solubility $W_s [%]$, the molar mass $M_s [kg/mol]$, the density $D_s [kg/m^3]$, the RHD($T_o$) $[\text{ref. temp.}]$ at reference temperature $T_o = 298.15 [K]$ and the corresponding temperature coefficients, $T_{cof}(RHD) [-]$. For all salt compounds, $\nu_i$ has been pre-determined with the bi-section method by the procedure of solving Eq. (5b) of M2012 (Eq. A6 in Appendix A4), using measurements of $W_s$–RHD($T_o$) (single data pairs) at $T_o$, which are listed with $\nu_i$ in Table 1. The required RHD values, including $T$-coefficients, have been taken from ISORROPIA II for a consistent comparison (Fountoukis and Nenes, 2007). All other data of Table 1 have been taken from the CRC Handbook of Chemistry and Physics (2006).

- The EQSAM4clim algorithm starts with the assignment of two internal loop parameters: An outermost loop, considering e.g., vertical model levels, and an innermost vector loop for e.g., the longitude–latitude grid box that contains the input-data for a given time-step. Both loops are scalable and can be externally determined depending on the climate model set-up, e.g., to best match the cache of the compute nodes. The consideration is optional and can be controlled in the subroutine call. The actual computations are structured in blocks, which are fully sequential. Each computational block has its own vector loop with the compound specific logic around it, so that loops can be fully optimized by the compiler and iterations between different computational blocks are avoided. All computational blocks are within an outermost loop (for this version).

- The first two computational blocks (out of 15) assign the $T [K]$ and RH $[0-1]$ data, as well as the lumped cations and anion concentrations $[mol/m^3 (air)]$. Further, the total cation and anion concentration charge is computed and a logical switch for each compound is defined. Thus, we assume that a compound can be formed only, if the product of the required cation and anion concentration is non-zero. This compound specific flag is subsequently used to skip the computation of individual compounds, which may not be present at the considered model time step. The flag is applied to all computational blocks which have an outer compound loop. In case all cation and anion concentrations are zero, i.e., the total cation charge equals (or is below) $\text{REALZERO} = \text{tiny}(0, dp)$ (with $dp = \text{SELECTED_REAL_KIND}(12,307)$), none of the compounds could form and we therefore skip all computations. Depending on the EMAC model set-up and the number of compounds considered, this may happen for instance in remote locations such as the upper stratosphere. Both options are merely included to minimize the overall computational burden.

- To further minimize the CPU time, the next computational block (3) defines:
  - Whether solids are excluded or included, i.e., the hysteresis loop. The criteria depend on the presence of aerosol water of the previous time step. In case aerosol water is not present, the aerosol is assumed to be dry and the water uptake is calculated based on RH and RHD thresholds. Otherwise, gas-liquid partitioning is considered and the water uptake is calculated without RHD thresholds, assuming the aerosol phase to be metastable.
  - The domain that needs to be considered. Similar to ISORROPIA II, we consider a domain approach (Sec. 2.2). But the approach used here only depends
on the input concentration ratio of total cations (tCAT) to total sulfate anions (tSO$_4^{2-}$), which is more elementary compared to the domain approach used in ISORROPIA II and is similar to the one used in the original EQSAM code (Metzger et al., 2002). Table 2 lists the domains used to characterize the potential sulfate aerosol neutralization levels.

- The next two computational blocks initialize the internal arrays, including all output fields, while block (5) defines the neutralization reaction order for all compounds that may form for a given domain during the cation-anion neutralization. The compound’s indices are ranked according to a preferred neutralization (from left to right). The domain dependent neutralization order is shown in Table 3.

- Computational block (6) solves the compound’s solute molality $\mu_s$ from Eq. (A3), by optionally including the Kelvin-term. To avoid iteration, we take a two-step approach / approximation (see Appendix B):
  - Step one: $K_e = 1$, and $B = 0$ to obtain the initial $\mu_s$ from Eq. (A3).
  - Step two, repeated three times: $\mu_s$ from previous iteration is used to calculate $K_e$ from Eq. (A7), $\chi_s$ from Eq. (A1), $B$ from Eq. (A4). Then a new $\mu_s$ is obtained from Eq. (A3).

Note that we sequentially solve the equations three times, whereby we only loop over those compounds of Table 1 that are allowed to form and those compounds that have a non-zero input ion-pair concentration (determined by block 2).

- The next computational block (7) calculates the temperature dependency of the RHD from Eq. (A5) and optionally considers the Kelvin effect according to Eq. (A7).

- In computational block 8, the actual cation-anion neutralization reactions are solved assuming chemical equilibrium. Based on the pre-defined reaction order the input cation and anion concentrations are balanced (neutralized) by looping over all compounds those cation-anion product is above REALZERO. As a result, the cation and anion concentrations subsequently decrease in favor of the concentrations of the corresponding compounds, independent of any solvent and solute activity. Within the same loop, we calculate the total solute activity and store the RHD values only for the compound’s that have a non-zero concentration. This information is subsequently used to analytically solve the liquid-solid partitioning.

- Computational block 9 approximates the mixed solution RHD from Eqs. (18-21).

- Thus far, all compounds have been treated as non-volatile and are assumed to reside in the aqueous phase. Computational block (10) solves the gas-liquid or gas-solid partitioning for the two semi-volatile compounds, NH$_4$NO$_3$ and NH$_4$Cl, that may be present in the sulfate neutral / poor (D1) domain (Sec. 2.4). Within one loop, Eqs. (2-5) are computed and the reactions (Eq. R2) are sequently solved. In case the RH is below the semi-volatile compound’s RHD, Eq. 1 is used.

- The liquid-solid partitioning is calculated for all salt compounds in computational block 11 from the weighted mixd solution approach, Eqs. (13-21), described in Sec. 2.6. Each compound is treated as solid (instantaneously precipitated from the
solution) in case the RH is below the compound’s RHD (binary solution), or in case of mixed solutions below the weighted RHD.

- Within computational block 12 all partial aerosol water mass are calculated from Eq. (22) for those compounds with a non-zero aqueous phase concentration. The total water mass is obtained from the sum of all partial water masses (Sec. 2.7).

- Computational block 13 estimates the final $\text{H}^+$ concentration [$\text{mol}$] from the difference of the total anion and cation concentrations. Within EMAC/GMXe the $\text{H}^+$ concentration is recalculated for both EQSAM4clim and ISORROPIA II to account for the changes in the aerosol precursor gas concentrations, which may result from the size-dependent condensation of $\text{HNO}_3$, $\text{HCl}$ and $\text{NH}_3$ and $\text{H}_2\text{SO}_4$ on the pre-existing aerosol surfaces (see Pringle et al., 2010a).

- Finally, the residual gases are calculated within computational block 14 from the remaining ion concentrations based on the implicit assumptions that: (i) Unneutralized $\text{NH}_4^+$ will instantaneously be fully vaporized to yield $\text{NH}_3$, unneutralized $\text{NO}_3^-$ yields $\text{HNO}_3$ and unneutralized $\text{Cl}^-$ yields $\text{HCl}$. (ii) In addition, unneutralized $\text{SO}_4^{2-}$ is assumed to yield $\text{H}_2\text{SO}_4$, which is however treated as non-volatile; vaporization of $\text{H}_2\text{SO}_4$ is considered within GMXe (Brühl et al., 2012). $\text{H}_2\text{SO}_4$ contributes to the water uptake (assuming the solute molality of $(\text{NH}_4)_3\text{H}(\text{SO}_4)_2$).

- The last computational block (15) prepares the model output, which is user specific and can be individually extended or configured to write out all aerosol properties.

Figure S1: EQSAM4clim computational algorithm (code length overview, no details).
Figure S2.1: EQSAM4clim flowchart.

Figure S2.2: EQSAM4clim flowchart (B9): Calculation of mixed solution RHD (Sect. 2.6).
S3 Extended Applications

This section extends/complements the results shown in Sec. 3:

1. Fixed solute concentrations (9 cases): ISORROPIA II and E-AIM
   (see main text, Sec. 3.1)
   (see main text, Sec. 3.2)
3. Variable solute concentrations (20 cases): ISORROPIA II and EQUISOLV II
   (see main text, Sec. 3.3)
4. Field observations (MINOS campaign, 184 cases): ISORROPIA II
   (see main text, Sec. 3.4)

S3.1 Fixed solute concentrations

Figure S3 extends the aerosol water mass calculations shown in Figure 4 (see Sec. 3.1) to 9 binary and mixed solution cases with fixed aerosol concentrations. The results are based on the full gas-liquid-solid partitioning and compare the calculations of EQSAM4clim with ISORROPIA II and E-AIM, with each (dry) compound concentration fixed to 1 \( \mu \text{mol/m}^3(\text{air}) \). The panels of Figure S3 show (from left to right, top-down):

- single solute solutions:
  (1.) NaCl; (2.) \((\text{NH}_4)_2\text{SO}_4\); (3.) \(\text{NH}_4\text{NO}_3\);
- mixed solutions:
  (4.) \(\text{NH}_4\text{HSO}_4 / (\text{NH}_4)_3\text{H(SO}_4)_2\);
  (5.) \(\text{NaHSO}_4 / \text{Na}_3\text{H(SO}_4)_2\);
  (6.) \(\text{NH}_4\text{NO}_3 - (\text{NH}_4)_2\text{SO}_4\);
  (7.) \(\text{NaNO}_3 - \text{NaCl} / \text{NaCl}\);
  (8.) \((\text{NH}_4)_2\text{SO}_4 - \text{NH}_4\text{Cl} - \text{Na}_2\text{SO}_4\);
  (9.) \(\text{NH}_4\text{NO}_3 - (\text{NH}_4)_2\text{SO}_4 - \text{NH}_4\text{Cl} - \text{Na}_2\text{SO}_4\)

The large panels show the aerosol water mass predictions for the RH range = 50 – 97 [%], while the small inserted panels expand the range to RH = 95 – 99.5 [%].
Figure S3: Extension of Figure 4 (main text): The bulk comparison of the total aerosol water mass, \( m_{w,mix} \) [kg/m\(^3\)(air)] obtained by Eq. (22) for EQSAM4clim, is shown for various single and mixed solutions. The dry concentration of each compound is fixed to 1 [\( \mu \text{mol}/\text{m}^3\)(air)] at \( T = 298.15 \) K. Results of EQSAM4clim (red crosses) and ISORROPIA II (green squares) are shown for \( RH = 50 - 97 \) [%] (large panels) and for the \( RH = 95 - 99.5 \) [%] (small panel). The results of E-AIM (web version) (blue circles) are included for comparison. The mixed solution RHD has been obtained for EQSAM4clim from Eq. (13–22) and are based on measured MDRH values for ISORROPIA II. The mutual deliquescence range of EQSAM4clim and ISORROPIA II (described in Sec. 2.6) differ from those of AIM (web version: http://www.aim.env.uea.ac.uk/aim/aim.php). Each panel is shown in the following for better reading.
Figure S3.1: Panel 1 of Figure S3 (Supplement).

Figure S3.2: Panel 2 of Figure S3 (Supplement).
Figure S3.3: Panel 3 of Figure S3 (Supplement).

Figure S3.4: Panel 4 of Figure S3 (Supplement).
Figure S3.5: Panel 5 of Figure S3 (Supplement).  

Figure S3.6: Panel 6 of Figure S3 (Supplement).
Figure S3.7: Panel 7 of Figure S3 (Supplement).

Figure S3.8: Panel 8 of Figure S3 (Supplement).
**S3.2 Variable \(NH_3\) concentration – SP2006**

Figure S4 extends the Fig. 5 (main text) to bi-sulfate and sulfuric acid. Both, the gaseous uptake of \(NH_3\) and \(HNO_3\) on saturated solutions and the weak dissociation of, e.g., \(H_2SO_4\) and \(HSO_4^-\), are not considered for EQSAM4clim (see Sec. S2). Therefore differences occur for bi-sulfate, sulfate and water in the concentration range of ammonia, i.e., within \(2 - 4 \mu g/m^3(air)\). At lower ammonia concentrations, where the sulfates are less neutralized, the bi-sulfate concentration increases and the sulfate concentration accordingly decreases, until only free sulfuric acid exists. Note that the differences between EQSAM4clim and ISORROPIA II are for ammonia concentrations below \(2 \mu g/m^3(air)\) only a matter of naming definition – the version of ISORROPIA II used considers all unneutralized sulfate simply as sulfate, an output variable sulfuric acid does not exist, since sulfuric acid has such a low vapor pressure that it practically only exists in the aerosol phase. EQSAM4clim has an option to treat it either way.
Figure S4: Extension of Figure 5 (main text): Mixed solution composition of $NH_4NO_3$ and $(NH_4)_2SO_4$ as a function of total ammonia at $T = 298.15 \ [K]$ and $RH = 70 \ [%]$, as defined in SP2006 for their Figure 10.23. $[TS] = [TN] = 10 \ [\mu g/m^3(air)]$ showing EQSAM4clim (red crosses) and ISORROPIA II (green squares). Each panel is shown in the following for better reading.
Figure S4.1: Panel 1 of Figure S4 (Supplement).

Figure S4.2: Panel 2 of Figure S4 (Supplement).
Figure S4.3: Panel 3 of Figure S4 (Supplement).

Figure S4.4: Panel 4 of Figure S4 (Supplement).
Figure S4.5: Panel 5 of Figure S4 (Supplement).

Figure S4.6: Panel 6 of Figure S4 (Supplement).
S3.3 Variable solute concentrations (20 cases)

Extension of Figure 6 and 7 to 20 aerosol composition cases. Cases 1–5 refer to the sulfate very rich regime, cases 6–10 to sulfate rich, and 11–20 to sulfate neutral and poor regimes (see Sec. 2.2). The concentrations of all aerosol components only depend on fixed molar ratios with respect to the total sulfate concentration, which is fixed to 20 [μg/m³(air)] for all 20 cases. The ratios are shown in Table 3 of Xu et al. (2009). Note that some of the cases are the same as in the model inter-comparison of Zhang et al. (2000), so that a direct comparison can be made to a wider range of equilibrium models, including AIM (the case number in the parenthesis in Table 3 of Xu et al., 2009, refers to the cases in the study by Zhang et al., 2000). Here, Figures S5–S7 show the corresponding results of EQSAM4clim, ISORROPIA II and EQUISOLV II for these 20 cases as a function of RH: 10, 20, 30, 40, 50, 60, 70, 80, 90, 95 [%]. The aerosol composition is calculated for each model from the gas-liquid-solid equilibrium partitioning, assuming deliquescence.

Figure S5 shows for the cases 1-20 (from left to right and top to bottom), the bulk aerosol water mass as a function of RH at $T = 298.15 \, [K]$. Fig. S6 shows the corresponding solid particulate matter (cases 11-20 in panels 1-10), panels 11-20 show the corresponding total dry particulate matter (PM), i.e., the sum of the liquid and solid aerosol mass (without aerosol water). Panels 1-10 of Fig. S7 show the total aerosol nitrate, while the panels 11-20 the aerosol ammonium concentration (both show cases 11-20).

Figure S5: Extension of Figure 6 (main text): Bulk aerosol water mass as a function of RH for various sulfate molar ratios, fixed for the entire RH range (at constant $T = 298.15 \, K$). Only the dry concentration ratio varies from case to case to match the domains of Table 2. The 20 aerosol composition cases refer to Table 3 of Xu et al. (2009). A subset of four panels is shown in the following for better reading.
Figure S5.1: Case 1-4 of Figure S5 (Supplement).

Figure S5.2: Case 5-8 of Figure S5 (Supplement).
Figure S5.3: Case 9-12 of Figure S5 (Supplement).

Figure S5.4: Case 13-16 of Figure S5 (Supplement).
Figure S5.5: Case 17-20 of Figure S5 (Supplement).

Figure S6: Extension of Figure 7 (main text): Total solid PM and liquid+solid PM.
Figure S6.1: Case 11-12 of Figure S6 (Supplement).

Figure S6.2: Case 13-14 of Figure S6 (Supplement).
Figure S6.3: Case 15-16 of Figure S6 (Supplement).

Figure S6.4: Case 17-18 of Figure S6 (Supplement).
Figure S7: Extension of Figure 7 (main text): Bulk aerosol nitrate and ammonium.
Nitrate and ammonium mass - case 11-12

Figure S7.2: Case 11-12 of Figure S7 (Supplement).

Nitrate and ammonium mass - case 13-14

Figure S7.3: Case 13-14 of Figure S7 (Supplement).
Figure S7.4: Case 15-16 of Figure S7 (Supplement).

Figure S7.5: Case 17-18 of Figure S7 (Supplement).
S3.4 Field observations – MINOS campaign

Extension of Figure 8 and 9 of Sec. 3.4. Figures S8 and Figure S9 show the gas-liquid-solid partitioning results of EQSAM4clim (red crosses) and ISORROPIA II (green squares). The equilibrium computations are based on lumped cation and anion concentrations, which were observed during MINOS in the aerosol fine and coarse mode, respectively.

Fig. S8, panels (1-20), show the model results for the aerosol fine mode (from left to right, top to bottom): total aerosol water mass [µg/m³], total particulate (aqueous+solid) matter (PM) [µg/m³], total solid PM [µg/m³], total (aqueous+solid) PM [µmol/m³], and in [nmol/m³] the (lumped) ion concentrations of: ammonium (NH₄⁺), sodium (Na⁺), potassium (K⁺), calcium (Ca²⁺), magnesium (Mg²⁺), sulfate (SO₄²⁻) – both as totals (aqueous+solid) and solids -, as well as total bi-sulfate (HSO₄⁻) and the residual gases, hydrochloric acid (HCl), nitric acid (HNO₃) and ammonia (NH₃).

Fig. S9, panels (1-20), show the model results for the aerosol coarse mode (from left to right, top to bottom): total aerosol water mass [µg/m³], total particulate (aqueous+solid) matter (PM) [µg/m³], total solid PM [µg/m³], total (aqueous+solid) PM [µmol/m³], and in [nmol/m³] the (lumped) ion concentrations of: ammonium (NH₄⁺), sodium (Na⁺), potassium (K⁺), calcium (Ca²⁺), magnesium (Mg²⁺), sulfate (SO₄²⁻), bi-sulfate (HSO₄⁻), nitrate (NO₃⁻), chloride (Cl⁻), – all both as totals (aqueous+solid) and solids, except ammonium and bi-sulfate, which are omitted because of their very low (negligible) concentrations. Figures S8 and S9 are enlarged below, by Fig. S8.1-S8.5 and Fig. S9.1-S9.5, respectively to show the details. Despite the different approaches in the mixed solution treatment of EQSAM4clim and ISORROPIA II, EQSAM4clim is relatively close to the results of ISORROPIA II, capturing many details of the solid precipitation of individual compounds for both, the fine and coarse mode.
Gas-liquid-solid Partitioning (fine mode)

\[ \text{Mg}^{2+}-\text{Ca}^{2+}-\text{Na}^{+}/\text{HCl/Cl-/NH}_3/\text{NH}_4^+/-\text{HNO}_3/\text{NO}_3^-/\text{H}_2\text{SO}_4/\text{HSO}_4/-\text{SO}_4^{2-}/\text{H}_2\text{O} \]

Figure S8: Extension of Figure 8 (main text): Aerosol water, total particulate matter and total solids \(\mu g/m^3(air)\), the corresponding residual gases \(\mu mol/m^3(air)\), and various ions for the fine mode. EQSAM4clim (red crosses), ISORROPIA II (green squares). A subset of four panels is shown in the following for better reading.

Gas-liquid-solid Partitioning (coarse mode)

\[ \text{Mg}^{2+}-\text{Ca}^{2+}-\text{Na}^{+}/\text{HCl/Cl-/NH}_3/\text{NH}_4^+/-\text{HNO}_3/\text{NO}_3^-/\text{H}_2\text{SO}_4/\text{HSO}_4/-\text{SO}_4^{2-}/\text{H}_2\text{O} \]

Figure S9: Extension of Figure S8 and Fig. 8 (main text) to the coarse mode.
Figure S8.1: Details of Figure S8 (Supplement): Aerosol water (aq), total mass (aq+s), lumped Na\(^+\)(aq+s) and K\(^+\)(aq+s) (nano moles).
Figure S8.2: Details of Figure S8 (Supplement): solid mass(aq+s), total moles(aq+s), lumped Ca\textsuperscript{2+}(aq+s) and Mg\textsuperscript{2+}(aq+s) (nano moles).
Figure S8.3: Details of Figure S8 (Supplement): Lumped NH$_4^+$ (aq+s), HSO$_4^-$ (aq+s), SO$_4^{2-}$ (aq+s) and Mg$^{2+}$ (s) (nano moles).
Gas-liquid-solid Partitioning (fine mode)

Minos observation period (28.07. - 22.08.2001)

Figure S8.4: Details of Figure S8 (Supplement): Lumped NH$_4^+$ (s), Na$^+$ (s), SO$_4^{2-}$ (s) and HCl(g) (nano moles).
Figure S8.5: Details of Figure S8 (Supplement): Lumped K\(^+\)(s), Ca\(^{2+}\)(s), HNO\(_3\)(g) and NH\(_3\)(g) (nano moles).
Figure S9.1: Details of Figure S9 (Supplement): Aerosol water(aq), total mass (aq+s), lumped Na⁺(aq+s) and K⁺(aq+s) (nano moles).
Figure S9.2: Details of Figure S9 (Supplement): solid mass(aq+s), total moles(aq+s), lumped Ca$^{2+}$(aq+s) and Mg$^{2+}$(aq+s) (nano moles).
Figure S9.3: Details of Figure S9 (Supplement): Lumped $\text{NH}_4^+$ (aq+s), $\text{HSO}_4^-$ (aq+s), $\text{SO}_4^{2-}$ (aq+s) and $\text{Ca}^{2+}$(s) (nano moles).
Figure S9.4: Details of Figure S9 (Supplement): Lumped \(\text{NO}_3^-(aq+s), \text{Cl}^-(aq+s), \text{Mg}^{2+}(s)\) and \(\text{SO}_4^{2-}(s)\) (nano moles).
Figure S9.5: Details of Figure S9 (Supplement): Lumped Na\textsuperscript{+}(s), K\textsuperscript{+}(s), NO\textsubscript{3}\textsuperscript{−}(s) and Cl\textsuperscript{−}(s) (nano moles).