Modelling and assimilation of lidar signals over Greater Paris during the MEGAPOLI summer campaign

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Abstract. In this study, we investigate the ability of the chemistry transport model (CTM) POL-LAIR3D of the air quality modelling platform POLYPHEMUS of simulating lidar backscattered profiles from model aerosol concentration outputs. To do so, simulated lidar signals are compared to hourly lidar observations performed during the MEGAPOLI (Megacities: Emissions, urban, regional and Global Atmospheric POLlution and climate effects, and Integrated tools for assessment and mitigation) summer experiment in July 2009, where a ground-based mobile lidar was deployed around Paris on-board a van. The comparison is performed for six different measurement days, 01, 04, 16, 21, 26 and 29 July 2009, corresponding to different levels of pollution and different atmospheric conditions. POLYPHEMUS correctly reproduces the vertical distribution of aerosol optical properties and their temporal variability. In the second part of this study, two new algorithms for assimilating lidar observations are presented. The aerosol simulations without and with lidar data assimilation are evaluated using the Airparif (a regional operational network in charge of air quality survey around the Paris area) data base to demonstrate the feasibility and the usefulness of assimilating lidar profiles for aerosol forecasts.

1 Introduction

Aerosols are key air quality species to monitor and model as they impact vegetation and as they impact human health by penetrating the respiratory system and leading to respiratory and cardiovascular diseases (Lauwerys et al., 2007; Dockery and Pope, 1996). They also impact visibility (Wang et al., 2009), and they represent an uncertain component of climate changes due to their effects on
the Earth’s radiative budgets (Intergovernment Panel on Climate Control (IPCC), 2007). For air quality, in order to simulate and predict particle concentrations, modellers have developed various chemistry transport models (CTM) in the past several years, e.g. EMEP (European Monitoring and Evaluation Programme) (Simpson et al., 2003), LOTOS (Long Term Ozone Simulation) - EUROS (European Operational Smog) (Schaap et al., 2004), CHIMERE (Hodzic et al., 2006), DEHM (Danish Eulerean Hemispheric Model) (Brandt et al., 2007) and POLYPHEMUS (Sartelet et al., 2007).

However, the aerosol vertical distribution is poorly quantified, because of numerous uncertainties on their sources (direct emissions) and on processes affecting their formation, e.g. nucleation, condensation, evaporation, and coagulation, as well as on meteorological conditions. As aerosol lifetime ranges from 1 to 10 days (Seinfeld and Pandis, 1998), improvements in the representation of their vertical distribution may lead to improved surface concentrations (lower error and higher correlation against observations) (Wang et al., 2013).

Various measurement types have been used to evaluate models. The most frequently used data are *in situ* surface measurements, e.g. AirBase (http://www.eea.europa.eu/) and EMEP over Europe, BDQA (Base de Données de la Qualité de l’Air) (Sartelet et al., 2007; Konovalov et al., 2009). However, they do not provide direct information on vertical profiles.

Satellite passive remote sensors (e.g. the Moderate Resolution Imaging Spectroradiometers (MODIS)) and sun-photometer surface stations (e.g. the AERosol RObotic NETwork (AERONET)) have greatly enhanced our ability to evaluate models. Comparisons between observed and simulated Aerosol Optical Thickness (AOT) have been implemented for global models and regional models (Kinne et al., 2006; Tombette et al., 2008; Péré et al., 2010). However, instruments, such as sun photometers can only retrieve column-integrated aerosol properties and can only work during daytime.

As accurate vertical profiles of aerosols can be measured by aerosol lidars, lidar measurements were used in several campaigns, for example to evaluate the transport of particles (Chazette et al., 2012). Moreover, aerosol lidar networks, such as the European Aerosol Research Lidar Network (EARLINET), are being developed at *in situ* sites. In space, measurements are performed with the Cloud-Aerosol Lidar with Orthogonal Polarisation (CALIOP) lidar (Winker et al., 2007). Ladar measurements have been used for the validation of aerosol models. For example, Hodzic et al. (2004) compared vertical profiles simulated by CHIMERE with those observed by lidars, from EARLINET, and Stromatas et al. (2012) used observations from the CALIOP space-based lidar. Royer et al. (2011) used an optical-to-mass relationships (urban, pre-urban and rural) to retrieve the PM$_{10}$ (particulate matter with diameter less than 10 $\mu$m) concentrations from lidar signals (Raut et al., 2009a,b). In Royer et al. (2011), lidar-derived PM$_{10}$ concentrations were compared with simulations from POLYPHEMUS and CHIMERE during the MEGAPOLI (Megacities: Emissions, urban, regional and Global Atmospheric POLLution and climate effects, and Integrated tools for assessment and mitigation) summer experiment in July 2009.

This paper is devoting to evaluating the lidar signal simulated by POLYPHEMUS during the MEGAPOLI
summer experiment, when a ground-based mobile lidar (GBLM) was deployed around Paris on-board a van. Measurements from a ground-based in-situ lidar at Saclay were also performed on 01 July 2009. This evaluation can also be regarded as a preprocessing stage of data assimilation (validation of the observation operator).

Data assimilation (DA hereafter) can reduce the uncertainties in input data such as initial or boundary conditions by coupling models to observations (Bouttier and Courtier, 2002). In air quality, applications of DA to \( \text{PM}_{10} \) forecast using in situ surface measurements have been performed by Denby et al. (2008) and Tombette et al. (2009) over Europe, and Pagowski et al. (2010); Pagowski and Grell (2012); Li et al. (2013) over the United States of America. Over Europe, the efficiency of assimilating lidar measurements to improve \( \text{PM}_{10} \) forecast has been compared to the efficiency of assimilating in situ surface measurements by Wang et al. (2013). They suggested that the assimilation of lidar observations may be more efficient to improve \( \text{PM}_{10} \) forecast, although it depends on the number of lidar stations used. However, Wang et al. (2013) did not directly assimilate the lidar signal, but they used a relation between mass concentration and optical properties of pollution aerosol. Although this kind of relation has been determined for pollution aerosols over Greater Paris (Raut et al., 2009a), it needs to be generalised to other measurement sites before operationally assimilating the mass concentration converted from the lidar signal. Moreover, the uncertainty linked to the estimation of mass concentrations may be about 25% (Raut et al., 2009a) mostly due to uncertainties in estimating the specific cross sections. Because uncertainties in the lidar signal may be less than 5%, it is more accurate to directly assimilate lidar signals.

This paper is organised as follows. Section 2 describes the experiment setup, i.e. the chemistry transport model used (POLYPHEMUS) and the observations. In section 3, the lidar observation operator is presented. Section 4 describes the evaluation of the simulation with in situ surface measurements and AERONET data. Results of the comparisons between observed and simulated lidar signals are shown in section 5. A new algorithm for the assimilation of lidar observations and results are shown in section 6. The findings are summarised and discussed in section 7.

2 Experiment setup

2.1 Presentation of the model

In this study, the POLAIR3D air quality model (Sartelet et al., 2007) of the air quality platform POLYPHEMUS, available at http://cerea.enpc.fr/polyphemus/ and described in Mallet et al. (2007), is used to simulate air quality over the Greater Paris area. Aerosols are modelled using the Size-RESolved Aerosol Model (SIREAM-SuperSorgam), which is described in Debry et al. (2007) and Kim et al. (2011). SIREAM-SuperSorgam includes 20 aerosol species: 3 primary species (mineral dust, black carbon and primary organic species), 5 inorganic species (ammonium, sulfate, nitrate, chloride and sodium) and 12 organic species. Five bins logarithmically distributed over the size
range 0.01 \(\mu m\) - 10 \(\mu m\) are used. The chemical mechanism CB05 (Carbon Bond version 5) is used for the gas chemistry (Yarwood et al., 2005). POLAIR3D/SIREAM has been used for several applications. For example, it was compared to in situ surface measurements for gas and aerosols over Europe by Sartelet et al. (2007, 2012); Couvidat et al. (2012), over Greater Paris by Couvidat et al. (2013), it was compared to AERONET data over Europe by Tombette et al. (2008) and to satellite data by Zhang et al. (2013), and it was compared to lidar-derived PM\(_{10}\) over Greater Paris during MEGAPOLI by Royer et al. (2011).

2.2 Modelling setup and observational data

The modelling domain is the same as the one used in Royer et al. (2011); Couvidat et al. (2013). It covers the Greater Paris area \([1.2^\circ E, 3.5^\circ E] \times [47.9^\circ N, 50.1^\circ N]\) with a horizontal resolution of 0.02° × 0.02°. Because Royer et al. (2011) show that limited vertical model resolution leads to much smoother vertical profiles than those deduced from lidar signals, a finer vertical resolution is used with twenty three vertical levels from the ground to 12000 m, instead of nine vertical levels in Royer et al. (2011). The simulations are carried out for one month from 28 June to 30 July 2009.

Meteorological inputs are the same as in Couvidat et al. (2013). They are simulated with the Weather Research & Forecasting (WRF) model (Skamarock et al., 2008) using an urban canopy model and an undated Corine land-use data base (Kim, 2011) with the YSU parameterisation (Hong et al., 2006) for the planetary boundary layer (PBL) dynamics. Anthropogenic emissions of gases and aerosols are generated with the Airparif (the Paris air quality agency) inventory for the year 2005. Boundary conditions for gaseous and particulate species were obtained from nested simulations over Europe and France, presented by Couvidat et al. (2013).

The ground based mobile lidar (GBML) used during the MEGAPOLI campaign is based on an ALS450 lidar commercialised by the LEOSPHERE company and initially developed by the Commissariat à l’Energie Atomique (CEA) and the Centre National de la Recherche Scientifique (CNRS) (Chazette et al., 2007). It provides lidar measurements at 355 nm. The main characteristics of this lidar are detailed in Royer et al. (2011). This system is particularly well-adapted to air pollution and tropospheric aerosol studies thanks to its full overlap reached at about 150-200 m height and its high vertical resolution of 1.5 m. Measurement days of 01, 04, 16, 21, 26 and 29 July 2009, which correspond to different levels of pollution from Airparif (low, moderate or high), are used for comparisons to the lidar signal. Moreover, ground-based in-situ lidar measurements were performed at Saclay on 01 July 2009 from 06:49 to 16:44 UTC 01 July 2009 (the blue square in Fig. 1). These measurements are used for both the comparison and the assimilation of lidar observations.

Airparif is the regional operational network in charge of air quality survey around the Paris area. It provides hourly gases and/or aerosol concentrations (PM\(_{10}\) and PM\(_{2.5}\)) measurements. Figure 1 shows the location of the Airparif stations with red squares and/or the magenta triangles. There are 17 stations at which PM\(_{10}\) and/or PM\(_{2.5}\) concentration measurements are performed.
The AERONET (AErosol RObotic NETwork) program is a federation of ground-based remote sensing aerosol networks established by NASA and PHOTONS (Univ. of Lille 1, CNES, and CNRS-INSU), which provides a long-term, continuous and readily accessible public domain database of aerosol optical measurements performed by sun-photometers. Sun-photometers measure AOD at different wavelengths ranging from 340 to 1024 nm. AOD data are computed for three data quality levels: Level 1.0 (unscreened), Level 1.5 (cloud-screened), and Level 2.0 (cloud-screened and quality-assured). The uncertainty of AOD measurements is less than 0.02 (Holben et al., 2001). For this study, there are 2 available stations over Greater Paris: Paris (urban station) and Palaiseau (suburban station) (the green discs in Fig. 1). In this paper, Level 2.0 AOD data at 340 and 380 nm are used to derive AOD data at 355 nm following the Ångström law:

\[
\text{AOD}(355) = \text{AOD}(340) \left(\frac{355}{340}\right)^{-\alpha},
\]

where \(\alpha\) is the Ångström exponent defined by

\[
\alpha = \ln \left(\frac{\text{AOD}(340)}{\text{AOD}(380)}\right) / \ln \left(\frac{380}{340}\right).
\]

3 Methodology

This section presents the methodology used in POLYPHEMUS to derive the lidar observation operator. The range-corrected lidar signal \(PR_2\) measured at an altitude \(z\) is defined by Collis and Russell (1976):

\[
PR_2(z) = C(\beta_m(z) + \beta_a(z)) \exp \left( -2 \int_0^z (\alpha_m(z') + \alpha_a(z')) dz' \right),
\]

where \(\beta_m\) (resp. \(\beta_a\)) is the molecular (resp. aerosol) backscatter coefficient, \(\alpha_m\) (resp. \(\alpha_a\)) is the molecular (resp. aerosol) extinction coefficient, and \(C\) is the instrumental constant for each channel depending on the technical characteristics of the emitting and receiving optics. In order to eliminate the instrumental constant \(C\) (because it is unknown), \(PR_2\) is normalised as follows

\[
H(z) = \frac{PR_2(z)}{PR_2(z_{\text{ref}})} = \frac{\beta_m(z) + \beta_a(z)}{\beta_m(z_{\text{ref}}) + \beta_a(z_{\text{ref}})} \exp \left( 2 \int_{z_{\text{ref}}}^z (\alpha_m(z') + \alpha_a(z')) dz' \right),
\]

where \(z_{\text{ref}}\) is taken at an altitude in the molecular zone. In equation (4), to estimate the normalised lidar signal \(H\), four optical parameters \(\beta_m, \beta_a, \alpha_m\) and \(\alpha_a\) are needed.

The molecular backscatter coefficient (\(\beta_m\)) at the wavelength \(\lambda\) of the incident light is calculated by Nicolet (1984):

\[
\beta_m = \frac{P}{k_B T} \cdot s_{\text{Ray}},
\]

where \(P\) is the pressure, \(T\) is the temperature, \(k_B\) is the Boltzmann constant, and the Rayleigh scattering cross section \(s_{\text{Ray}}\) is given by

\[
s_{\text{Ray}} = 4.678 \cdot 10^{-29} \cdot \lambda^{-3.916+0.074 \cdot \lambda + 0.05/\lambda}.
\]
The molecular extinction coefficient ($\alpha_m$) is given by Nicolet (1984)

$$\alpha_m = \frac{8 \pi}{3} \beta_m,$$  \hspace{1cm} (7)

Aerosol extinction and backscatter coefficients ($\beta_a$ and $\alpha_a$) are functions of particle sizes, of the aerosol complex refractive index (ACRI) of particles $m$, and of the wavelength $\lambda$ of the incident light. With a population of different-sized particles of identical refractive index $m$ and with a number size distribution function $n(D_{wet})$ with $D_{wet}$ the particle wet diameter, the aerosol extinction and backscatter coefficients are given by the following formulas:

$$\alpha_a = \int_0^{D_{wet}^{max}} \frac{\pi D_{wet}^2}{4} Q_{ext}(m,a_{wet})n(D_{wet})dD_{wet},$$  \hspace{1cm} (8)

and

$$\beta_a = \int_0^{D_{wet}^{max}} \frac{\pi D_{wet}^2}{4} Q_{bsca}(m,a_{wet})n(D_{wet})dD_{wet},$$  \hspace{1cm} (9)

where $D_{wet}^{max}$ is a wet diameter upper limit for the particle population, $a_{wet} = \frac{\pi D_{wet}}{\lambda}$ a dimensionless size parameter, $Q_{ext}(m,a_{wet})$ and $Q_{bsca}(m,a_{wet})$ are extinction and backscatter efficiencies. These efficiencies are computed through the Mie code (de Rooij and van der Stap, 1984; Mishchenko et al., 2002) from ftp://ftp.giss.nasa.gov/pub/crmim/spher.f. The dry complex refractive index (CRI) is interpolated from the OPAC package (Hess et al., 1998) for each species at the desired wavelength $\lambda$ (355 nm). The CRI and densities used for calculation of optical properties are shown in Table 1. The wet diameter $D_{wet}$ is computed from the mean dry diameter of each section of the aerosol sectional model SIREAM and from the aerosol water content. The aerosol water content is calculated from the thermodynamic model ISORROPIA (Nenes et al., 1998).

Computing the ACRI requires to make an assumption on the mixing state of the aerosol chemical species. The current version of POLYPHEMUS is based on an assumption of aerosol internal mixing: all the particles of a given size section at a given grid point of the domain are supposed to have the same chemical composition. Within this framework, Tombette et al. (2008) compared aerosol optical properties using 2 different assumptions for the Black Carbon (BC) mixing state: internally homogeneous mixing and core-shell mixing. In the internally homogeneous mixing case, BC is treated as the other components and a volume-weighted ACRI is calculated from the CRI of pure species. In the core-shell mixing case, each particle is assumed to have a structure: the core (BC) and the shell (all the other components). The hypothesis of an internally homogeneous mixing state seems to be unphysical as BC can not be well-mixed in the particle because of its complex geometry and solid state (Katrinak et al., 1993; Sachdeva and Attri, 2007). Tombette et al. (2008) have shown that the use of these two mixing states leads to negligible differences on AOD, but non-negligible differences on single scattering albedo and absorption process, in agreement with illustrations of Jacobson (2000). Thus, a core-shell mixing hypothesis is used in this study. The Maxwell-Garnet approximation is used to calculate ACRI from the core CRI (i.e., BC in this study) and the shell CRI (where all the other components are well mixed).
3.1 Estimation of $z_{ref}$

The altitude used to normalise the lidar signal does not need to correspond exactly to the beginning altitude of the molecular zone, but it could be any arbitrary chosen altitude in the molecular zone, where there is almost no aerosol. However, it is better to use an estimation of the normalisation altitude as close as possible to the beginning of the molecular zone, because lidar signals are attached to higher uncertainties at high altitudes. Although the molecular zone is often determined visually from lidar vertical profiles, this method is not efficient to treat large amounts of lidar profiles. We therefore created a new algorithm which can automatically estimate the normalisation altitude $z_{ref}$ from the lidar vertical profile.

The normalisation altitude $z_{ref}$ is estimated from the lidar signal and the simulated molecular signal $S_{Ray}$

$$S_{Ray}(z) = \beta_m(z) \exp \left( -2 \int_0^z \alpha_m(z')dz' \right), \quad (10)$$

as follows:

- Define a weight for each vertical point of the lidar signal (the vertical resolution is 1.5 m). The weights should be larger for the points that are more likely to be in the molecular zone, i.e. at high altitudes. We used $w(h) = \exp((h-h_{max})/L)/L$, where $h$ is the altitude of the points, $h_{max}$ is the maximal altitude considered (e.g. 4 km) and the parameter $L$ is taken equal to 200 m.

- Fit all lidar signal vertical points (noted as a vector $y$) with a weighted least absolute deviations (LAD) regression (DasGupta and Mishra, 2007). In detail, we minimise

$$\| (y - (ah+b))^T w \|_{L1} = \sum_i |w_i(y_i - ah_i - b)| \quad (11)$$

to find $a$ and $b$ (cyan lines in Fig. 2).

- Calibrate the simulated molecular signal $S_{Ray}$ with the LAD regression line at altitude $h_{max}$, and calculate the difference between the calibrated $S_{Ray}$ and the LAD regression line at each vertical point of the lidar signal in a loop starting from high altitudes to low altitudes. The altitude at which the difference becomes larger than a pre-assigned value (1% of the value corresponding to the LAD regression line) corresponds to $z_{ref}$.

Figure 2 shows comparisons between the lidar signal and the simulated molecular signal $S_{Ray}$ for different lidar measurement days during MEGAPOLI. The simulated molecular signal (red lines in Fig. 2) agrees well with the lidar observations (blue points in Fig. 2) at high altitudes in the molecular zone, leading to the determination of the molecular zone and $z_{ref}$. 
4 Model evaluation

To evaluate air quality models, Boylan and Russell (2006) recommended PM model performance goal and criterion that are based upon an analysis of numerous PM and visibility modelling studies. The PM model performance goal corresponds to the level of accuracy that is considered to be close to the best a model can be expected to achieve. The PM model performance criterion corresponds to the level of accuracy that is considered to be acceptable for modelling applications. The Mean Fractional Bias (MFB) and the Mean Fractional Error (MFE) are proposed to evaluate model performances against observations. RMSE and correlation are also often used in the aerosol modelling community. The statistical indicators are defined in appendix A.

4.1 Model evaluation with Airparif data

Table 2 shows statistics for the month of simulation and for the 6 lidar measurement days. For the month of simulation, for PM$_{2.5}$, the MFB and MFE are respectively in the range [-30 %, 30%] and [0, 50 %], i.e. the PM model performance goal is met. For PM$_{10}$, the MFB and MFE are respectively in the range [-60 %, 60%] and [0, 75 %], i.e. the PM model performance criterion is met. For each lidar measurement day, the PM model performance goal is always met for PM$_{2.5}$, and the PM model performance criterion is met for PM$_{10}$ except for 29 July.

As shown in Table 2, the model simulates well PM$_{2.5}$ concentrations, but PM$_{10}$ concentrations are underestimated. In other words, coarse particles (particulate matter with a diameter higher than 2.5 µm and lower than 10 µm) are underestimated. This may be because emissions and boundary conditions of coarse particles are underestimated, e.g. road resuspensions of PM is not considered in the model and boundary conditions are obtained from nested simulations over Europe and France where coarse particles were underestimated.

4.2 Model evaluation with AERONET data

Table 3 presents statistics for hourly data: the simulation underestimates the AOD, in agreement with the result of the comparisons between the simulation and Airparif observations (see section 4.1). The simulated and the observed AOD agree well on 01, 04, 16 and 26 July 2009, according to the criteria of Boylan and Russell (2006). However, the correlation is only -8% on 01 July, which is a day of high pollution (see Table 2). To understand why the correlation is low on 01 July, Figure 3 presents the hourly evolution of the PM$_{10}$ concentration (resp. simulated AOD) against Airparif (resp. AERONET) surface measurements at the station “Paris”. At the surface, there is a good agreement (90.38 % correlation) between the simulation and Airparif observations for the PM$_{10}$ concentration on 01 July 2009. However, the hourly simulated AOD and the surface measured PM$_{10}$ do not correlate well with AERONET data at this station. Although the surface PM$_{10}$ concentration starts to increase from 02:00 UTC, the AOD only begins to increase from 09:00 UTC. It means that
between 02:00 and 09:00 UTC, PM$_{10}$ concentration is high near the surface but not at high altitudes. However, in the model, the AOD increases from as early as 05:00 UTC. A possible explanation of the discrepancy between observed and simulated AOD is that the particles are mixed by turbulence more effectively and earlier in the model than in the true state of the atmosphere on 01 July (Wang et al., 2013). As shown in Fig. 3, there is a time difference (about 2 hours) between the peak of AOD and the peak of simulated/measured surface PM$_{10}$ concentrations, probably because a strong thermic mixing occurring in the late morning during this lidar measurement day (Royer et al., 2011).

5 Comparisons with lidar vertical profiles

The simulated lidar signal is compared with GBML observations performed during the MEGAPOLI summer experiment on the different measurements days (01, 04, 16, 21, 26 and 29 July 2009). The purpose of this section is to validate the ability of POLYPHEMUS to simulate lidar backscattered profiles and then choose suitable measurement days to do assimilation tests.

On 01 July 2009, GBML measurements are performed leeward inside the pollution plume in the Southwest of Paris between Saclay and Chateaudun during 3 hours (the black track in Fig. 1). It is the most polluted day of the MEGAPOLI experiment. High levels of PM$_{10}$, on average about 45 µg m$^{-3}$ (see Tab. 2), are measured by the Airparif network. Figure 4 presents the comparison between lidar observations and the simulation at 11:00, 12:00 and 13:00 UTC. It shows that POLYPHEMUS underestimates the lidar signal at 13:00 UTC, but it overestimates it at 14:00 UTC and it agrees well with observations at 15:00 UTC. While the boundary layer height increases from about 1.2 to 1.8 km from 11:00 to 13:00 UTC and the GBML runs out of the pollution plume (Royer et al., 2011), both the observed and simulated lidar signals decrease. Figures of the comparison between the simulation and observations from a ground-based in-situ lidar at Saclay are shown later in this paper. The pollution plume covers Saclay because of the Northeast wind. Thus high lidar signal values in both the simulation and observations are seen after 10:00 UTC, although the simulated lidar signals are underestimated. Data assimilation will be performed for this day, as it is the most polluted day with observations from both the GMBL and a ground-based in-situ lidar.

On 04 July 2009, GBML measurements are performed around Paris with a circular pattern from 14:49 to 17:24 UTC. Particle AOD and concentrations are underestimated in the simulation. The daily averaged AOD from the AERONET network is about 0.25, respectively 0.14 in the simulation (see Tab. 3). The daily averaged PM$_{10}$ concentration from the Airparif network is about 18.37 µg m$^{-3}$, respectively 11.11 µg m$^{-3}$ in the simulation (see Tab. 2). Figure 5 shows the comparison between the GBML measurements and the simulation at 15:03 UTC and 16:00 UTC. The simulated lidar signals are underestimated. Moreover, lidar measurements show an aerosol layer between 2.0 km and 3.0 km (probably from long-range transport), which is not present in the simulation.

On 16 July 2009, GBML measurements are performed in the North of Paris from Saclay to
Amiens between 11:00 UTC and 14:30 UTC. The lidar signal is overall underestimated, as shown in Fig. 6, in agreement with the underestimation of PM$_{10}$ shown by the statistics in Tables 2 and 3. Surface PM$_{10}$ concentration from the Airparif network and from the simulation are respectively 26.25 and 16.47 µg m$^{-3}$ (low-moderate level of pollution, see Tab. 2). The observed and simulated AOD are respectively 0.26 and 0.18 (see Tab. 3). The simulated AOD has a good correlation with AERONET data (up to 80 %). As deduced from the comparisons of the modelled and observed lidar signals in Fig. 6, the PBL height is well modelled until 12:00 UTC, but it is underestimated afterwards, e.g. the PBL height is about 2.1 km from the observed lidar signal but it is about 1.6 km in the simulation. These differences in PBL height explain that the simulated lidar signal agrees better with the observation until 12:00 UTC.

On 21 July 2009, the GBML travels from Saclay to the North of Paris across the city centre of Paris. As shown in Fig. 7, the lidar signal is overestimated for this measurement day. However, the surface PM$_{10}$ concentration is underestimated. It is 27.84 and 16.84 µg m$^{-3}$ (low-moderate level of pollution, see Tab. 2) from the Airparif network and from the simulation respectively. The large simulated lidar signals originate in high aerosol concentration at high altitudes, i.e. between 2.0 km and 2.5 km, which leads to higher backscattering and extinction coefficients. This high-altitude aerosol layer originates in boundary conditions, but it is not present in the observations. It impacts the lidar signal until low altitudes. This is why surface PM$_{10}$ is underestimated while lidar signal is overestimated.

On 26 July 2009, the GBML followed two circular patterns (the yellow and cyan tracks in Fig. 1). One is performed from 12:40 to 15:30 UTC at a distance between 15 and 30 km from the city centre. Another one is performed from 16:44 to 18:18 UTC in the South-Southwest of Paris. Low levels of pollution are observed and simulated. Surface PM$_{10}$ concentration and AOD are underestimated. The daily averaged PM$_{10}$ concentration from Airparif is 18.04 µg m$^{-3}$, against 10.12 µg m$^{-3}$ in the simulation. The mean observed AOD value is 0.15, against 0.08 in the simulation. Although the lidar signal is slightly underestimated in the simulation, simulated and observed lidar signals agree fairly well, as shown in Figure 8. The pollution from Paris is transported by the South wind to the North. This is why the lidar signal is higher at 14:00 UTC in Fig. 8. Because as much as 5 hours of lidar measurements are performed, which is longer than on 04, 16, 21 and 29 July 2009, we will perform data assimilation for this day.

On 29 July 2009, GBML measurements are performed from 12:22 to 15:10 UTC in the North of Paris and in peri-urban and rural areas. While low levels of pollution (12.33 µg m$^{-3}$ of the mean PM$_{10}$ concentration in Tab. 2) are simulated, moderate levels of pollution (29.25 µg m$^{-3}$ of the mean PM$_{10}$ concentration in Tab. 2) is observed by the Airparif network. As deduced from Figure 9, at the beginning of measurement period, the PBL height is about 1.5 km and the simulated lidar signal agrees well with lidar observations. At 15:00 UTC, the observed lidar signal has increased, because of an aerosol layer between 2.0 and 3.5 km. This layer is not simulated and the simulated
lidar signal is underestimated.

6 Assimilation test of lidar observations

As mentioned in the previous section of comparisons between the simulation and the lidar observations during the MEGAPOLI summer experiment in July 2009, DA run is performed for 01 (5 hours of measurements) and 26 (13 hours of measurements) July 2009.

In air quality, the large number of state variables leads to high computational costs when implementing DA algorithms. Among the widely used DA algorithms, the optimal interpolation (OI) is used here, as it is the most computationally efficient (Denby et al., 2008; Tombette et al., 2008; Wu et al., 2008; Li et al., 2013). In applications of DA to aerosol forecast, Tombette et al. (2009) have used the OI over western Europe for assimilating observations from the BDQA network, which covers France. Denby et al. (2008) have used two different DA techniques, the OI and EnKF, to assimilate PM$_{10}$ concentrations over Europe. Pagowski et al. (2010) have used the OI for multiple aerosol species and for prediction of PM$_{2.5}$ in the Los Angeles basin. And Wang et al. (2013) have used the OI over Europe to investigate the potential impact of future ground-based lidar networks on analysis and short-term forecasts of PM$_{10}$.

6.1 Basic formulation

The basic formulation of DA of lidar signals with OI is now described. Particles are represented in the model by mass concentrations of different chemical species for the different particle size sections.

The state vector $x$ is defined by

$$x = \{x^h_{i,j,k}\}_{1 \leq i \leq N_b, 1 \leq j \leq N_s, 1 \leq k \leq n, 1 \leq h \leq l}, \quad (12)$$

where $x^h_{i,j,k}$ is the mass concentration of the aerosol species $j$ in section $i$ for the horizontal spatial grid $k$ at the model vertical level $h$, $N_b$ is the number of size sections, $N_s$ is the number of chemical species, $n$ is the number of horizontal grid points at each vertical level $h$ and $l$ is the total number of vertical levels. The lidar observation operator is $H(x) = L \cdot S(x)$, where $S$ is a nonlinear operator from the model state $x$ to the lidar signal state, and $L$ is a linear spatial interpolation operator.

The analysed state vector is a solution to the variational optimisation problem:

$$x^a = \text{Argmin}_x J(x), \quad (13)$$

where $J$ is the cost function defined by

$$J(x) = \frac{1}{2} (H(x) - y)^T R^{-1} (H(x) - y) + \frac{1}{2} (x - x^b)^T B^{-1} (x - x^b)$$

$$\simeq \frac{1}{2} (H(x^b) + L S(x - x^b) - y)^T R^{-1} (H(x^b) + L S(x - x^b) - y)$$

11
\[
\frac{1}{2} (x - x^b)^T B^{-1} (x - x^b),
\]

where \( S \) is the tangent linear of operator \( S \), \( B \) and \( R \) are the matrices of error covariances for backgrounds and observations respectively, and \( y \) is the vector of observations. In this way, we have

\[
\nabla J(x^a) = (LS)^T R^{-1} (H(x^b) - y) + (B^{-1} + (LS)^T R^{-1} (LS)) (x^a - x^b) = 0,
\]

which leads to

\[
x^a - x^b = (B^{-1} + (LS)^T R^{-1} (LS))^{-1} (y - H(x^b))
\]

\[
= B (LS)^T ((LS)B(LS)^T + R)^{-1} (y - H(x^b)).
\]

### 6.2 Construction of error covariances

As the measurements at different levels originate from the same lidar, the matrix \( R \) should not be diagonal because of measurement error correlations. However, in order to simplify \( R \) in the first tests of DA of lidar observations, one takes \( R = r I \) as a diagonal matrix where \( I \) is the identity matrix and \( r \) is an error variance. The value of the observation error variance \( r \) is determined by a \( \chi^2 \) diagnosis (Ménard et al., 1999), in which the scalar

\[
\chi^2 = (y - H(x^b))^T ((LS)B(LS)^T + R)^{-1} (y - H(x^b))
\]

should be equal, on average, to the number of observations \( N \) at each DA step.

Specifically, \( B \) plays a role in determining how the corrections of the concentrations should be distributed over the domain during DA. In practice, however, it is impossible to accurately know all coefficients of \( B \). In our simulation, the number of model grid points is of the order of \( 10^5 \). Thus the number of coefficients in the matrix \( B \) is about \( 10^{10} \) multiplied by the square of the number of analysis variables (about 100 variables for particles are used here). Therefore, \( B \) is too large to be handled numerically.

In order to reduce the size of the error covariance matrices for background, we model the matrix \( B \) as follows

\[
B = PDP^T,
\]

where \( D \) is the error covariance matrix for \( \text{PM}_{10} \), defined by the Balgovind approach (Balgovind et al., 1983) obtained by considering the RMSE and correlation of simulated \( \text{PM}_{10} \) concentrations. Thus, the size of \( D \) is much less than the one of \( B \). The matrix \( P \) is defined by

\[
P = \begin{bmatrix}
v_1 & 0 & \ldots & 0 \\
0 & v_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & v_M \\
\end{bmatrix}_{(M \cdot N_b \cdot N_s) \times M},
\]
where $M$ is equal to the dimension of the domain $(l \cdot n)$, $\mathbf{v}_k$ is a vector of size $N_b \cdot N_s$ (the number of state variables). Each component of $\mathbf{v}_k$ corresponds to the proportion of the mass of particles for a given species in a given size section in PM$_{10}$ mass concentrations at grid point $k$.

Let $\mathbf{S}' = \mathbf{SP}$ be the directional derivative of $S$ along a given direction, and let $c^b$ and $c^a$ be PM$_{10}$ concentration states before and after analysis respectively. We multiply each side of equation (17) by the matrix $\mathbf{Z}$ in order to convert $x$ into the PM$_{10}$ state $c$:

$$
\mathbf{Z} = \begin{bmatrix}
1 & \ldots & 1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\
0 & \ldots & 0 & 1 & \ldots & 1 & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \ldots & 0 & 0 & \ldots & 0 & 1 & \ldots & 1
\end{bmatrix}_{M \times (M \cdot N_b \cdot N_s)}
$$

We obtain

$$
c^a - c^b = \mathbf{D} (\mathbf{LS}')^T \left( (\mathbf{LS}') \mathbf{D} (\mathbf{LS}) + \mathbf{R} \right)^{-1} (\mathbf{y} - H(\mathbf{x}^b)).$$

(20)

After the analysis, the concentrations $c^a$ are redistributed over particle species and size sections following the initial chemical and size distributions.

### 6.3 DA setup

DA experiments are carried out for 01 and 26 July 2009. All DA experiments are performed with time step 600 s and from 200 to 1800 m above the ground (10 model levels), since the lidar measurements are not available below the altitude of full overlap (200 m above the ground) and since aerosol concentrations above the PBL have limited impact on surface PM$_{10}$ in the short term (Wang et al., 2013). In the Balgovind approach, the horizontal correlation length is set to 0.2$, which is estimated from numerical DA tests. The error variances are separately set for each DA level, depending on the RMSE of PM concentrations and the variability of PM concentrations at each model level.

Two new algorithms are tested for the assimilation of lidar observations. In the first algorithm, we use the assimilation of lidar observations to analyse PM$_{10}$ concentrations and the analysed PM$_{10}$ concentrations are redistributed over particle species and size sections following the initial chemical and size distributions (see section 6.2). The background error variances ($\mathbf{PM}_{b}$) are estimated by the simulation without DA and Airparif observations. The value of the observation error variance $r$ is determined by a $\chi^2$ diagnosis, which yields $r = 1$ $\mu g^2$ m$^{-6}$ and $r = 0.006$ $\mu g^2$ m$^{-6}$ respectively for 01 and 26 July, depending on the level of uncertainties (see section 5). Let $N$ be the number of lidar observations at one DA step. Figure 10 shows the time evolution of $\chi^2/N$ (blue lines) for DA runs on 01 and 26 July. The mean over DA window of $\chi^2/N$ is 1.02 (resp. 1.02) for 01 (resp. 26) July.

In the second algorithm, we separately analyse PM$_{2.5}$ and PM$_{2.5-10}$ (particulate matter with a diameter higher than 2.5 $\mu m$ and lower than 10 $\mu m$) in the assimilation of lidar observations. We modify the matrices used in section 6.2 to obtain $c_{2.5}$ and $c_{2.5-10}$, the mass concentrations of PM$_{2.5}$ and PM$_{2.5-10}$ respectively (see Appendix B for details). We separately set the error variances
for PM$_{2.5}$ and PM$_{2.5-10}$ in matrix $D$. Because of the lack of PM$_{2.5-10}$ observations, we cannot directly estimate the background error variances. They are determined by the $\chi^2$ diagnosis using the observation error variance $r$ found in the first algorithm.

In the following, we note the assimilation with the first (resp. second) DA algorithm as “DA (PM$_{10}$)” (resp. “DA (PM$_{2.5}$ and PM$_{2.5-10}$)”).

### 6.4 Results and discussions

In these DA tests, the purpose is to verify if these new algorithms are functional. Because we work on a small scale, the corrections of DA are transported out of the simulation domain very quickly. Thus we only compute the statistics for the DA window to validate the DA tests.

Table 4 presents statistics of the simulation results without DA and with DA. Statistics are computed for both PM$_{10}$ and PM$_{2.5}$ concentrations. Overall, both DA algorithms lead to better scores (lower RMSE, MFB and MFE, and higher correlation) than the simulation without DA for PM$_{10}$ concentrations. Comparing two DA algorithms, the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$) leads to better scores than the simulation with DA (PM$_{10}$) for PM$_{10}$ concentrations (see Tab. 4). The RMSE of PM$_{10}$ is 11.63 $\mu g \cdot m^{-3}$ in the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$), against 13.69 $\mu g \cdot m^{-3}$ in the simulation with DA (PM$_{10}$) on 01 July. The RMSE of PM$_{10}$ is 4.73 $\mu g \cdot m^{-3}$ in the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$), against 6.08 $\mu g \cdot m^{-3}$ in the simulation with DA (PM$_{10}$) on 26 July. It is because higher background error variances are set for the coarse sections in the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$). However, the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$) leads to similar scores to the simulation with DA (PM$_{10}$) for PM$_{2.5}$ concentrations (see Tab. 4). It is because similar background error variances for PM$_{2.5}$ in the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$) to the simulation with DA (PM$_{2.5}$) are used in the $\chi^2$ diagnosis, since fine particles contribute to more than 80% of the lidar signal (Randriamiarisoa et al., 2006). In the following, we compare the simulation without DA and the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$).

On 01 July, the averaged RMSE of PM$_{10}$ is 11.63 $\mu g \cdot m^{-3}$ with DA (PM$_{2.5}$ and PM$_{2.5-10}$), against 17.74 $\mu g \cdot m^{-3}$ without DA. The decrease of the RMSE are explained by the correlation length in the matrix $D$, since no Airparif station performs measurements in the Southwest of Paris (the Northeast wind). At station **ISSY-LES-MOULINEAUX**, the closest station to Saclay, the RMSE of PM$_{10}$ is 14.72 $\mu g \cdot m^{-3}$ with DA (PM$_{2.5}$ and coarse), against 22.81 $\mu g \cdot m^{-3}$ without DA. However, the averaged RMSE of PM$_{2.5}$ is about 10.4 $\mu g \cdot m^{-3}$ with DA (PM$_{2.5}$ and PM$_{2.5-10}$), against 8.54 $\mu g \cdot m^{-3}$ without DA. This is due to the larger horizontal correlation length (see section 6.3). Figure 11 shows that the model underestimates the lidar signal at Saclay. While DA runs increase PM concentrations in the lidar measurement grids, PM concentrations are increased at Airparif stations, where PM$_{2.5}$ concentrations is well simulated and coarse particles are underestimated. This problem can be solved by decreasing the horizontal correlation length.

On 26 July, the averaged RMSE of PM$_{10}$ is 4.73 $\mu g \cdot m^{-3}$ with DA (PM$_{2.5}$ and PM$_{2.5-10}$), against
6.67 µg m$^{-3}$ without DA. Because two circular GBML travelling patterns were performed around Paris (see Fig. 1), most of Airparif stations are leeward (the South wind) or they are close to the patterns of GBML. They could validate improvements of PM concentrations. At station *PARIS l'er Les Halles*, the RMSE of PM$_{10}$ is 1.96 µg m$^{-3}$ in the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$), against 4.71 µg m$^{-3}$ in the simulation without DA. Moreover, DA runs lead to better scores than the simulation without DA for PM$_{2.5}$. At leeward station *CREIL FAIENCERIE*, the RMSE of PM$_{2.5}$ is 4.1 µg m$^{-3}$ in the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$), against 4.9 µg m$^{-3}$ in the simulation without DA.

### 7 Conclusions

In order to investigate the ability of the CTM POLAIR3D of the air quality modelling platform POLYPHEMUS to simulate lidar vertical profiles, we have performed a simulation in the Greater Paris area for the summer month July 2009. The results (PM$_{10}$ and PM$_{2.5}$ concentrations) are evaluated by Airparif data. We have simulated aerosol optical properties and lidar signals from the model aerosol concentration outputs using the aerosol complex refractive index (ACRI) and the wet particle diameter. Hourly comparisons between simulated lidar signals and lidar observations have been described for six measurement days during the MEGAPOLI summer campaign. These comparisons have shown a good agreement between GBML measurements and the simulation except 21 July 2009, where an aerosol layer was presented at higher altitude in the model. The results show that the optical property module of POLYPHEMUS would reproduce correctly lidar signals in the model, if the aerosol layer is well simulated.

Two new algorithms for the assimilation of lidar observations have been presented. That depends on whether PM$_{10}$ is analysed or PM$_{2.5}$ and PM$_{2.5-10}$ are both analysed. DA tests were performed for 01 and 26 July 2009. On the whole, both of these algorithms lead to better scores (lower RMSE, MFB and MFE, and higher correlation) for PM$_{10}$. However, they did not work for PM$_{2.5}$ on 01 July 2009, because of the larger horizontal correlation length. The simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$) leads to better scores than the simulation with DA (PM$_{10}$) by setting separately the error variances for backgrounds in fine sections and coarse sections. The results shown in this paper suggest that the assimilation of lidar observations for analysing PM$_{2.5}$ and PM$_{2.5-10}$ would perform better than assimilating the lidar signal for analysing PM$_{10}$, but it is computationally more costly.

Comparing the simulation without DA and the simulation with DA (PM$_{2.5}$ and PM$_{2.5-10}$), the averaged RMSE of PM$_{10}$ is 11.63 µg m$^{-3}$ with DA (PM$_{2.5}$ and PM$_{2.5-10}$), against 17.74 µg m$^{-3}$ without DA on 01 July 2009. The averaged RMSE of PM$_{10}$ is 4.73 µg m$^{-3}$ with DA (PM$_{2.5}$ and PM$_{2.5-10}$), against 6.67 µg m$^{-3}$ without DA on 26 July 2009.

A forthcoming paper will present results about the assimilation of continuous measurements from the ACTRIS/EARLINET network during a 72-hour period of intensive observations.
Acknowledgements. This work was supported by CEA (Commissariat à l’Énergie Atomique) and CEREA, joint laboratory École des Ponts ParisTech – EDF R & D. This paper is also a contribution to the INSU/LEFE ADOMOCA-2 project. The authors thank their colleague Florian COUVIDAT for his help to perform the simulation used in this paper. The authors would like to thank Philippe ROYER and Jean Christophe RAUT for their contributions in the MEGAPOLI summer campaign.

Appendix A Statistical indicators

\{o_i\}_{i=1,n} and \{s_i\}_{i=1,n} are the observed and the modelled concentrations at time \(i\), respectively. \(n\) is the number of available observations. The statistical indicators used to evaluate the results with respect to observations are: the Root Mean Square Error (RMSE), the (Pearson) correlation, the Mean Fractional Error (MFE), the Mean Fractional Bias (MFB). MFE and MFB bound the maximum error and bias and do not allow a few data points to dominate the statistics. They are often used to evaluate model performances against observations for aerosol mass concentrations and optical properties (Boylan and Russell, 2006). The statistical indicators are defined as follow:

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (o_i - s_i)^2}, \tag{A1}
\]

\[
\text{correlation} = \frac{\sum_{i=1}^{n} (o_i - \bar{o})(s_i - \bar{s})}{\sqrt{\sum_{i=1}^{n} (o_i - \bar{o})^2 \sum_{i=1}^{n} (s_i - \bar{s})^2}}, \tag{A2}
\]

\[
\text{MFE} = \frac{1}{n} \sum_{i=1}^{n} \frac{|s_i - o_i|}{(s_i + o_i)/2}, \tag{A3}
\]

\[
\text{MFB} = \frac{1}{n} \sum_{i=1}^{n} \frac{s_i - o_i}{(s_i + o_i)/2}, \tag{A4}
\]

where \(\bar{o} = \frac{1}{n} \sum_{i=1}^{n} o_i\) and \(\bar{s} = \frac{1}{n} \sum_{i=1}^{n} s_i\).

Appendix B Update formula for DA (PM\textsubscript{2.5} and PM\textsubscript{2.5−10})

In order to separately analyse PM\textsubscript{2.5} and PM\textsubscript{2.5−10} in the assimilation of lidar observations, the matrix \(B\) is modelled as follows

\[
B = PD^TP^T, \tag{B1}
\]

where \(D\) is the error covariance matrix for PM\textsubscript{2.5} and PM\textsubscript{2.5−10}. The matrix \(D\) is defined by

\[
D = \begin{bmatrix}
D\textsubscript{2.5} & 0 \\
0 & D\textsubscript{2.5−10}
\end{bmatrix},
\]

and the matrix \(P\) is defined by

\[
P = \begin{bmatrix}
P\textsubscript{2.5} \\
P\textsubscript{2.5−10}
\end{bmatrix}.
\]
where each column $k$ of $P_{2.5}$ (resp. $P_{2.5-10}$) corresponds to the proportion of the mass of particles for a given species in a given size section in PM$_{2.5}$ (resp. PM$_{2.5-10}$) mass concentrations at grid point $k$ as section 6.2 shown.

The matrix $Z$ is defined by

$$Z = \begin{bmatrix} Z_{2.5} \\ Z_{2.5-10} \end{bmatrix},$$

where the matrix $Z_{2.5}$ (resp. $Z_{2.5-10}$) is a $M \times (M \cdot N_b \cdot N_s)$ matrix, which converts the state vector $x$ into the PM$_{2.5}$ (resp. PM$_{2.5-10}$) state $c_{2.5}$ (resp. $c_{2.5-10}$).

Let $S' = SP$. After multiplying each side of equation (17) by the matrix $Z$, we obtain

$$\begin{pmatrix} c_{2.5}^a - c_{2.5}^b \\ c_{2.5-10}^a - c_{2.5-10}^b \end{pmatrix} = D(LS')^T \left( (LS')D(LS')^t + R \right)^{-1} (y - H(x^b)).$$


Mallet, V., Quelo, D., Sportisse, B., Ahmed de Biasi, M., Debray, E., Korsakissok, I., Wu, L., Roustan, Y,


Fig. 1. The blue square shows the location of the ground-based *in-situ* lidar station, the red squares (resp. the magenta triangles) show the locations of Airparif stations for PM$_{10}$ (resp. PM$_{2.5}$) measurements and the green discs show the locations of AERONET stations. The black pattern shows the GBML track on 01 July 2009. The yellow and cyan patterns show two GBML tracks on 26 July 2009. The rectangle area is detailed in the bottom figure.
Fig. 2. Blue points (resp. red lines) indicate lidar signals PR$_2$ (resp. simulated molecular signals $S_{Ray}$) at 13:00 UTC 01, 16:00 UTC 04, 12:00 UTC 16, 15:00 UTC 21, 14:00 UTC 26 and 14:00 UTC 29 July 2009 (blue points). LAD regressions of weighted lidar measurement points are indicated by cyan lines.
Fig. 3. Left (resp. right) figure shows the hourly evolution of the PM$_{10}$ concentration (resp. AOD) at station Paris for 01 July 2009.

Fig. 4. Comparisons between the vertical profiles observed by GBML (blue points) and simulated by POLYPHEMUS (red lines) on 01 July 2009 from 11:00 to 13:00 UTC. Lidar observations below the altitude of full overlap are not represented.
Fig. 5. Comparisons between the vertical profiles observed by GBML (blue points) and simulated by POLYPHE-MUS (red lines) on 04 July 2009 at 15:03 and 16:00 UTC. Lidar observations below the altitude of full overlap are not represented.

Table 1. Dry CRI and density for different aerosol species at $\lambda = 355$ nm. Re (resp. Im) stands for the real (resp. imaginary) part of CRI.

<table>
<thead>
<tr>
<th>Species</th>
<th>Re</th>
<th>Im</th>
<th>density (g cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrate</td>
<td>1.53</td>
<td>-0.005</td>
<td>1.5</td>
</tr>
<tr>
<td>Ammonium</td>
<td>1.53</td>
<td>-0.005</td>
<td>0.91</td>
</tr>
<tr>
<td>Black carbon</td>
<td>1.75</td>
<td>-0.4645</td>
<td>2.25</td>
</tr>
<tr>
<td>Mineral dust</td>
<td>1.53</td>
<td>-0.0166</td>
<td>2.33</td>
</tr>
<tr>
<td>Organics</td>
<td>1.53</td>
<td>-0.008</td>
<td>1.3</td>
</tr>
<tr>
<td>Sulfate</td>
<td>1.45</td>
<td>-1e-08</td>
<td>1.84</td>
</tr>
<tr>
<td>Sodium</td>
<td>1.509</td>
<td>-2.946e-07</td>
<td>0.97</td>
</tr>
<tr>
<td>Chlorate</td>
<td>1.509</td>
<td>-2.946e-07</td>
<td>1.15</td>
</tr>
<tr>
<td>Water</td>
<td>1.35738</td>
<td>2.72875e-08</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Fig. 6. Comparisons between the vertical profiles observed by GBML (blue points) and simulated by POLYPHE-MUS (red lines) on 16 July 2009 at 11:03, 12:00, 13:25 and 14:09 UTC. Lidar observations below the altitude of full overlap are not represented.

Table 2. Statistics (see Appendix A) of the simulation results for the Airparif network during the MEGAPOLI summer experiment. Obs. stands for observation. Sim. stands for simulation. Corr. stands for correlation.

<table>
<thead>
<tr>
<th>Day</th>
<th>PM$_{10}$</th>
<th>PM$_{2.5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs. mean</td>
<td>Sim. mean</td>
</tr>
<tr>
<td></td>
<td>µg m$^{-3}$</td>
<td>µg m$^{-3}$</td>
</tr>
<tr>
<td>All</td>
<td>21.53</td>
<td>14.14</td>
</tr>
<tr>
<td>01</td>
<td>44.99</td>
<td>29.39</td>
</tr>
<tr>
<td>04</td>
<td>18.37</td>
<td>11.11</td>
</tr>
<tr>
<td>16</td>
<td>26.25</td>
<td>16.47</td>
</tr>
<tr>
<td>21</td>
<td>27.84</td>
<td>16.84</td>
</tr>
<tr>
<td>26</td>
<td>18.04</td>
<td>10.12</td>
</tr>
<tr>
<td>29</td>
<td>29.25</td>
<td>12.33</td>
</tr>
</tbody>
</table>
Fig. 7. Comparisons between the vertical profiles observed by GBML (blue points) and simulated by POLYPHE-MUS (red lines) on 21 July 2009 at 12:15, 13:16, 14:10 and 15:10 UTC. Lidar observations below the altitude of full overlap are not represented.

Table 3. Statistics (see Appendix A) of the simulation results for the AERONET network for different lidar measurement days.

<table>
<thead>
<tr>
<th>Day</th>
<th>Obs. mean</th>
<th>Sim. mean</th>
<th>RMSE</th>
<th>Corr.</th>
<th>MFB</th>
<th>MFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.59</td>
<td>0.47</td>
<td>0.20</td>
<td>-8</td>
<td>-21</td>
<td>29</td>
</tr>
<tr>
<td>04</td>
<td>0.25</td>
<td>0.14</td>
<td>0.12</td>
<td>37</td>
<td>-58</td>
<td>58</td>
</tr>
<tr>
<td>16</td>
<td>0.26</td>
<td>0.18</td>
<td>0.08</td>
<td>80</td>
<td>-33</td>
<td>33</td>
</tr>
<tr>
<td>26</td>
<td>0.15</td>
<td>0.08</td>
<td>0.07</td>
<td>45</td>
<td>-53</td>
<td>53</td>
</tr>
</tbody>
</table>
Fig. 8. Comparisons between the vertical profiles observed by GBML (blue points) and simulated by POLYPHE-MUS (red lines) on 26 July 2009 at 13:00, 14:00, 15:00, 17:00 and 18:10 UTC. Lidar observations below the altitude of full overlap are not represented.
Fig. 9. Comparisons between the vertical profiles observed by GBML (blue points) and simulated by POLYPHE-MUS (red lines) on 26 July 2009 at 13:00, 14:00 and 15:00 UTC. Lidar observations below the altitude of full overlap are not represented.

Fig. 10. Time evolution of $\chi^2/N$ (blue lines) for DA runs on 01 and 26 July 2009. The mean over DA window of $\chi^2/N$ is 1.02 (resp. 1.02) for 01 (resp. 26) July 2009.
Fig. 11. Lidar vertical profiles observed by the ground-based *in-situ* lidar at Saclay (blue points), simulated without DA (red lines) and simulated with DA (magenta lines) on 01 July 2009.
Table 4. Statistics (see Appendix A) of the simulation results (PM$_{10}$ and PM$_{2.5}$) without DA and with DA for the Airparif network for 01 and 26 July 2009. “With DA (PM$_{10}$)” stands for the assimilation of lidar observations correcting directly PM$_{10}$. “With DA (PM$_{2.5}$ and PM$_{2.5-10}$)” stands for the assimilation of lidar observations correcting separately PM$_{2.5}$ and PM$_{2.5-10}$.

<table>
<thead>
<tr>
<th>Day</th>
<th>Species</th>
<th>Sim.</th>
<th>Obs. mean</th>
<th>Sim. mean</th>
<th>RMSE</th>
<th>Corr.</th>
<th>MFB</th>
<th>MFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>PM$_{10}$</td>
<td>Without DA</td>
<td>47.26</td>
<td>32.35</td>
<td>17.74</td>
<td>84</td>
<td>-41</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>With DA (PM$_{10}$)</td>
<td>36.20</td>
<td>13.69</td>
<td>90</td>
<td>-29</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>With DA (PM$<em>{2.5}$ and PM$</em>{2.5-10}$)</td>
<td>39.85</td>
<td>11.63</td>
<td>84</td>
<td>-19</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PM$_{2.5}$</td>
<td>Without DA</td>
<td>30.52</td>
<td>30.21</td>
<td>8.54</td>
<td>69</td>
<td>-5</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>With DA (PM$_{10}$)</td>
<td>33.04</td>
<td>10.44</td>
<td>59</td>
<td>5</td>
<td>27</td>
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<tr>
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<td>With DA (PM$<em>{2.5}$ and PM$</em>{2.5-10}$)</td>
<td>33.08</td>
<td>10.45</td>
<td>58</td>
<td>5</td>
<td>27</td>
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<tr>
<td>26</td>
<td>PM$_{10}$</td>
<td>Without DA</td>
<td>16.25</td>
<td>9.96</td>
<td>6.67</td>
<td>-20</td>
<td>-47</td>
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<td>With DA (PM$_{10}$)</td>
<td>10.55</td>
<td>6.08</td>
<td>15</td>
<td>-42</td>
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<td>With DA (PM$<em>{2.5}$ and PM$</em>{2.5-10}$)</td>
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<td>4.73</td>
<td>26</td>
<td>-25</td>
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<td>PM$_{2.5}$</td>
<td>Without DA</td>
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<td>8.99</td>
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<td></td>
<td>With DA (PM$<em>{2.5}$ and PM$</em>{2.5-10}$)</td>
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<td>2.54</td>
<td>21</td>
<td>-4</td>
<td>22</td>
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