Replies to referees' comments on ACPD 11, C8564–C8565, 2011: "A hybrid bin scheme to solve the condensation/evaporation equation using a cubic distribution function"

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1 First referee's comments

1.1

Referee

The use of Legendre polynomials is a common tool in discontinuous Galerkin methods for hyperbolic problems (cite for instance the introductory paper by Cockburn).

It is possible to perform the positivity tests and the redistribution procedure in the Legendre basis.

Authors

We agree it would be useful to provide a reference about the Legendre polynomials. We believe a standard reference about their role in approximation theory would be more useful than a paper about DG methods, so have cited the book by Dahlquist and Björck (1974). Please see Sect. 2.4 in the manuscript where the new reference is added.

Indeed the positivity test can be performed easily in the Legendre basis. However, doing so for the redistribution procedure is more complicated. This is because the grid divisions μ must be transformed into χ using Eq. (13) in the manuscript. Because m_0, m_ℓ and m_r in this equation are different for different bins, this transformation is not uniform across bins. We found this to be less convenient than performing the redistribution in the standard polynomial basis.

1.2

Referee

The phrase extra work should be comment with some numbers concerning the additional flops. The error measures should be explained more precisely. How is the root-mean-square error related to the L_1 error? Since the bins are not equally spaced in the first example the L_1 error should be a more appropriate measure. The error plots (Fig. 2 and Fig. 4) may be dominated by the time error for increasing number of bins. The computations should be repeated for a smaller time step. It should be mentioned that the error plots are half logarithmic.

Authors

Good point. The extra computing time required when the cubic scheme is implemented is shown in the (new) Sect. 3.3 in the revised manuscript.

We agree with the referee and switch to L_1 errors. The formula used to compute the errors is given by Eq. (29) in the revised manusript.

In Sect. 3.1 (the first numerical test), the time step is decreased from 1.0 s in the original manuscript to 0.1 s in the revised manuscript. Experiments with a time step less than 0.1 s in the first test (Sect. 3.1) and less than 1.0 s in the second test (Sect. 3.2) show little difference.

The logarithmic scale of the errors is indicated in the captions of Figs. 2 and 4 in the revised manuscript.

1.3

Referee

The initial and the exact solution should be presented as a continuous curve (use more points for plotting). The bin number axes should be avoided. Take the radius axes above. Indicate the type of plotting (linear or logarithmic). A comparison for finer grid resolution may be also useful. There is some feeling, that the chosen resolution is very crude (at least from a mathematical point of view). It is possible to make some order studies which confirms that the new interpolation leads to a better approximation order.

Authors

The initial and exact solutions have been interpolated into the coarse grid so that they can be compared directly with the numerical solutions. For this reason it is helpful to leave these curves at the same resolution as the numerical solutions.

We decided to retain both the radii and the bin numbers on the axes of Figs. 1 and 3. We think that the bin numbers are helpful for the readers to locate the solutions with respect to the grid.

Our cubic scheme is not, in fact, a higher order accurate approximation because we do not use the cubic sub-bin structure to compute the microphysical tendencies. In practice, bin schemes are not implemented at very high resolutions. Thus, improvements in accuracy by the cubic scheme must be demonstrated at relatively low resolutions, and the real-world advantages of the cubic scheme do not depend on it being higher-order. Please also see the last paragraph of Sect. 3.3 in the revised manuscript. There we briefly discuss the relevance of the cubic scheme to cloud resolving models that are in the intermediate range of resolutions.

1.4

Referee

The examples are simple by the chosen growth rate function. A test with the growth rate function mentioned in the appendix of the seminal paper by Chen and Lamb would be very interesting. Their non-monotonic behavior allows the simultaneously condensation and evaporation of different bins.

Authors

We agree that it is important to test the scheme in a variety of settings. Toward this end we have investigated the test suggested by the referee. We report on those results below. However, we felt this test yielded results similar to those already presented, and since it is most applicable to very small droplets during their activation state, it does not add much generality to our results. Therefore we have included different third test, involving a more complex two-dimensional simulation of thin cirrus clouds.

Here are the results for the Chen and Lamb problem. When the curvature and solute effects are included, the growth rate of cloud drops is

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{B}{r}\left(S - \frac{\alpha}{r} + \frac{\beta}{r^3}\right),$$

where r is the radius of cloud drops, S is the supersaturation ratio, and α, β and B are independent of r. The above growth rate is most applicable to droplets during the activation stage. For this test let $S = 0.01$, $\alpha = 1.2 \times 10^{-9}\,\mathrm{m}^{-1}$, $\beta = 1.5 \times 10^{-22}\,\mathrm{m}^{-3}$. There is no analytical solution for this case. The solutions obtained by the linear and cubic schemes at low resolution are compared with a numerical solution obtained by either scheme at very high solution (500 bins).

Let the initial drop spectrum be Eq. (24) in the manuscript, with $N_0 = 2.0 \times 10^8\,\mathrm{m}^{-3}$ and $m_c = 9.0 \times 10^{-16}\,\mathrm{kg}$ here. The latter is the mass of a drop of radius $0.6\,\mu\mathrm{m}$. The bin grid is defined according Eqs. (26)–(28), where $r_{\mathrm{min}} = 0.1\,\mu\mathrm{m}$ and $r_{\mathrm{max}} = 4\,\mu\mathrm{m}$.

For this case, special time discretization techniques (see Chen and Lamb (1994, Appendix)) are necessary to take a time step that is not impractically small. However, since this is a theoretical test for the discretization along the mass axis, we will simply use the forward Euler's time stepping method. This requires a very small time step $\triangle t = 5 \times 10^{-5}\,\mathrm{s}$.

The solutions at 1 s obtained by the linear and cubic schemes at 10 bins are shown here in Fig. 1. The errors in the solutions at 1 s obtained by the linear and cubic schemes as functions of bin resolution are shown in Fig. 2. Similar to other cases presented in the manuscript, the cubic scheme performs better than the linear scheme.

1.5

Referee

How often does the algorithm switch back to linear approximation?

Authors

We agree this should be discussed and it is now covered at the end of Sect. 3.1.3. In particular, in the evaporation problem (Sect. 3.1 in the manuscript), the cubic scheme switches from cubic to linear approximations in respectively 14 % and 4 % of the time at 20 and 80 bins. In the depositional growth problem (Sect. 3.2 in the manuscript), the cubic scheme switches from cubic to linear approximations in respectively 25 % and 4 % of the time at 6 and 20 bins. At higher resolutions the cubic approximation is used more often because the discretized numerical solution is smoother.

2 Second referee's comments

2.1

Referee

P. 21639, line 5: Values from this equation will not be within the range -1 and 1, thus violates the definition of χ on p. 21637, line 16. Will this cause a problem? If not, then the definition of χ should be modified.

Authors

Good point; χ should be defined such that it varies between -1 and 1 as m varies between m_ℓ and m_r. When $m = \overline{m}_L$, which is smaller than m_ℓ, χ is smaller than -1. When $m = \overline{m}_R$, which is larger than m_r, χ is greater than 1. Since values of χ may be outside the $[-1, 1]$ range, the sentence "χ is an independent variable defined between -1 and 1" is wrong. We have shortened it to "χ is an independent variable" in the revised manuscript.

2.2

Referee

P. 21643, Section 3.2: Mathematically, this case is the same as that presented in the previous section. The only difference is to allow supersaturation to vary with time (i.e. open system versus closed system), but this variation is of no numerical importance. Besides, ice crystal growth has its own complexity that comes from the shape effects. I suggest the authors focusing on the numerical methods and conduct more thorough analysis, such as comparing the results for case 1 with increasing resolutions (such as those given in Table 1). One should see the results to converge at high resolutions.

Authors

In addition to the difference pointed out by the referee, the bin grids are defined differently in the two tests. By showing the second test, we demonstrate the robust performance of the cubic scheme over different grids. Furthermore, in the first test, the numerical solutions are compared with an analytical solution. In the second test, there is no analytical solution because the growth rate of particles is more complex. In this case the numerical solutions at low resolutions are compared with a numerical solutions obtained at a very high resolution.

We elected to keep the second test because it serves as a transition to the third test in Sect. 3.3. The microphysics of ice crystals in the second test is similar to that in the third test. Sect. 3.3 is a new section which was not in the original manuscript.

The decrease in the errors as bin resolution increases (Figs. 2 and 4 in the revised manuscript) indicates that the two schemes converge at high resolution. We think that further analyses of the convergence of the schemes at very high resolutions are not necessary because this has little use in practice. Most cloud resolving models fall into the range of intermediate resolutions (less than 100 bins). For further discussions of this issue, please see the last paragraph of Sect. 3.3 in the revised manuscript.

2.3

Referee

P. 21645, lines 5-6: "For the same accuracy, the number of bins required for the cubic scheme is less than that of the linear scheme." The authors addressed only the computation accuracy but not the computation efficiency. A reduction of bin number by might not be much, considering that the cubic method necessarily use more computation time than the linear method on the individual bin bases. Does the reduced bin number (by) enough to compensate the extra computation? The authors need to provide a CPU-time analysis.

Authors

These are indeed important points. We added Sect. 3.3 into the revised manuscript to address the referee's concerns. In this new section, we evaluate the accuracy as well as the computing time of the linear and cubic schemes in a two-dimensional cloud resolving model. We have been using this model to simulate thin cirrus in the tropical tropopause layer.

In this model, there is a significant improvement in accuracy when the the cubic scheme is used. This is achieved at only 5 % increase in computing time. For our own research using this model, we are willing to pay the additional cost to achieve the improved accuracy.

Alternatively, the cubic scheme can be used at lower resolution than the linear scheme to save computing time. This is achieved without loss of accuracy. In the tests that we performed, the computing time saved by reducing the bin resolution outweighs the additional time required to run the cubic scheme.

2.4

Referee

P. 21645, Summary: Bin models also need to deal with collision processes. One of the key benefits of the cubic method is to reduce the number of bins for condensation/deposition calculation. But reducing the bin number necessarily causes large error in the collision growth processes, unless the cubic method can also be easily applied to such calculations. The authors discussed a little bit about this in the Summary section and provided a seemly good solution, i.e. use less bins with the cubic method for the condensation-dominant sizes, whereas apply more bins with the linear method for the collision-dominant sizes. However, the most critical collisions are raindrop initiation collisions between large cloud drops, which is still in the condensation-dominant size regime. Also, the condensation process tends to narrow the cloud drop spectrum, and often only very few bins (may be a little as 1 if the bin sizing factor is above 2) contain cloud drops. Yet, most bin models do not consider collisions within a bin. If only 1 bin contains cloud drops, there will be no collision in such models. So, decreasing the bin number in the condensation-dominant size regime inevitably causes artificial reduction in rain initiation by collision. Therefore, reducing the bin number is not necessary the best numerical strategy.

Authors

We agree that one cannot arbitrarily plan to reduce the bin resolution if one switches to the cubic scheme. We trust that microphysical modelers will use a variety of physical considerations to decide what the minimum bin resolution should be in each size range.

The reduction in resolution applies only when the particle spectrum is sufficiently resolved by the bin grid. If the spectrum of particles is contained within only one bin, the resolution is far too coarse to resolve the spectrum. In that case neither the linear nor the cubic scheme performs well. A brief discussion concerning the resolutions applicable to cloud resolving models is contained in the last paragraph of Sect. 3.3 of the revised manuscript.

We do not suggest reducing the number of bins over the size range in which collision is important. As indicated in the title of the manuscript, so far we have applied the cubic scheme to the evaporation/condensation problem only.

2.5 Minor comments

- *Referee:* P. 21633, line 17: "aggregation". The traditional term for drop-drop interaction is coalescence.

  Authors: We rewrote this sentence accordingly.

- *Referee:* p. 21635, line 6: "$\mu_1, \mu_2, \ldots, \mu_{N+1}$". Why not just use $m_1, m_2, ..., m_{N+1}$ as done in the previous page? If m_i are intended to be variable (i.e. Lagrangian boundaries), then modify the previous notation of m with m.

Authors: We use $\mu_1, \mu_2, \ldots, \mu_{N+1}$ for the grid divisions. For each bin resolution, i.e. for each N, the grid divisions are fixed, whereas the variable m may change over different time steps. m_1 and m_2 in the original manuscript denote the left and right boundaries of each bin. During the bin shift procedure, the bins are shifted with respect to the grid. Thus the grid divisions should be distinguished from the variable m. To eliminate this confusion, we decided to change m_1 and m_2 to m_ℓ and m_r.

- *Referee:* P. 21636: "the assumption that the growth rate of particles in the bin is equal to the growth rate of the mean mass is least accurate at the bin boundaries." It is better explaining this earlier on the previous page.

  Authors: We feel that this sentence is most appropriate there. This is where we discuss the best choice for the additional points necessary to rewrite the distribution from linear to cubic.

- *Referee:* P. 21637, line 20: How is this equation calculated for partially empty bins?

  Authors: We do not correct the distribution function for positivity when calculating the coefficients for the cubic polynomial. It is the correction for positivity that results in the bin being partially empty. This correction is performed after the cubic coefficients have been found and if the resulting cubic polynomial is negative in part of the bin.

- *Referee:* P. 21638, Eq. 16: Should the exponent in the denominator be 3 instead of 2?

  Authors: We checked the equation and think that the exponent in the denominator should be 2.

- *Referee:* P. 21642, line 20: RMS errors. Can you also show the RMS error for mass?

  Authors: As suggested by the first referee, we switched to L_1 errors in the revised manuscript. The L_1 errors for mass are now shown in the revised manuscript.

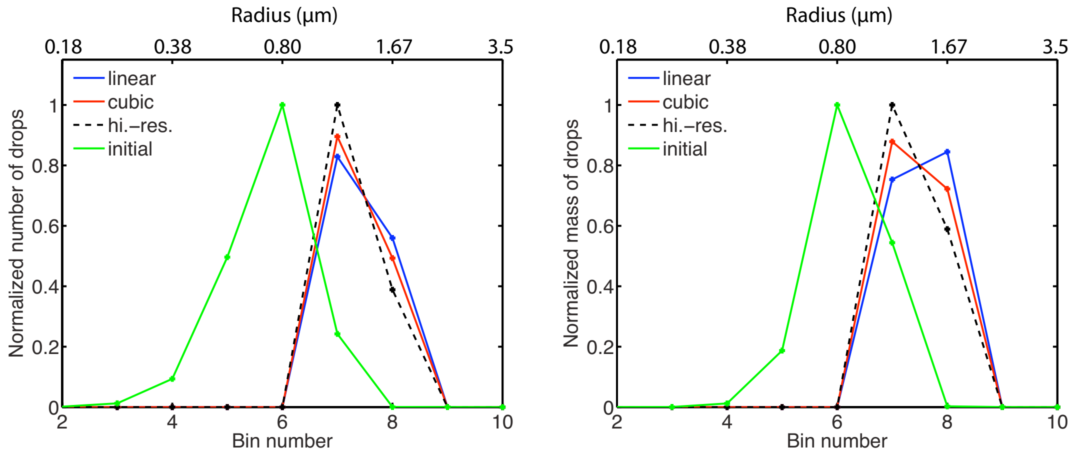
- *Referee:* With Figs. 2 and 4, Tables 1 and 2 are redundant.

  Authors: We agree and have removed the tables from the manuscript.

References

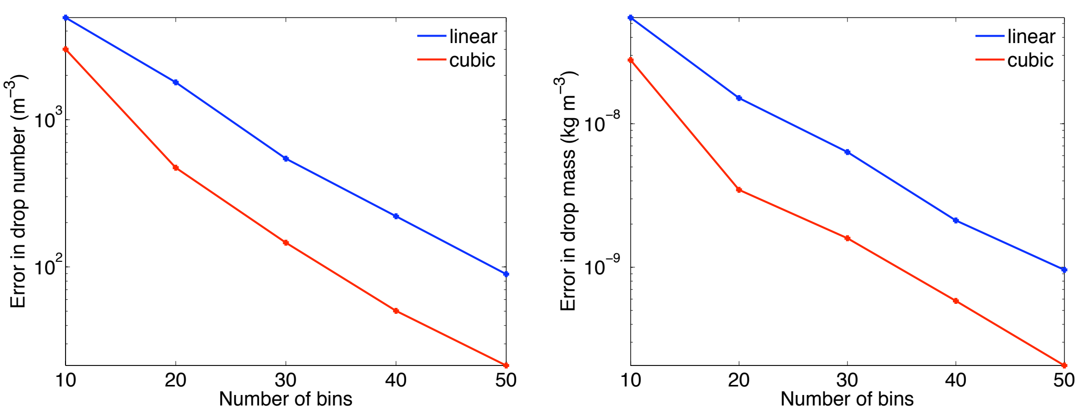
Chen, J. P. and D. Lamb, 1994: Simulation of cloud microphysical and chemical processes using a multicomponent framework. Part I: Description of the microphysical model. *J. Atmos. Sci.*, **51 (18)**, 2613–2613.

Dahlquist, G. and Å. Björck, 1974: *Numerical Methods*. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.



(a) (b)

Figure 1: Numerical solutions in (a) number and (b) mass of drops obtained by the linear and cubic schemes at 1 s in the droplet activation problem. The low resolution solutions (blue and red curves) are obtained at 10-bin-resolution. The high resolution solution (black, dashed curve) is obtained at 500-bin-resolution. The initial and final solutions are normalized by the maximum of the high resolution solutions at respectively the initial and final time. The radii corresponding to the masses at the bin centers are indicated at the top of the plot.



(a) (b)

Figure 2: L_1 errors in (a) number and (b) mass of drops of the solutions at 1 s obtained by the linear and cubic schemes in the droplet activation problem. The errors are plotted in logarithmic scale.