Interactive comment on “Where do winds come from? A new theory on how water vapor condensation influences atmospheric pressure and dynamics” by A. M. Makarieva et al.

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Condensation rate and hydrostatic equilibrium of moist air

In our paper (hereafter M10) we provided an expression for condensation rate $S$ (Eq. (34) in M10). This equation expresses condensation rate as the difference between (a) the total change of vapor density with height and (b) the density change caused by adiabatic expansion. Here we explore the physical meaning of this expression from a different perspective. We shall show that Eq. (34) corresponds to the statement that condensation rate $S$ is linear over the amount of $N_v$ of vapor (i.e., condensable gas) in the atmosphere. Using this relationship as an assumption, we shall show that $S(34)$ follows directly from the condition that the vertical distribution of moist air remains in equilibrium.

Using Eqs. (32), (33) and $\partial N_d/\partial x = 0$ of M10 we obtain:

$$u \frac{\partial N_d}{\partial x} = (S_d - S) \frac{1}{\gamma_d} \tag{1}$$

where

$$S_d \equiv w \left( \frac{\partial N_v}{\partial z} - \gamma_d \frac{\partial N_d}{\partial z} \right), \quad \gamma_d \equiv \frac{N_v}{N_d}. \tag{2}$$

The magnitude of condensation rate $S$ in (1) remains unknown. Note that under terrestrial conditions $1/\gamma_d \gg 1$. In a dry atmosphere, where condensation rate is zero, the distribution of air would be in equilibrium in both vertical and horizontal dimensions, such that $\partial N_d/\partial x \to 0$ at $N_v \to 0$.

We write the condition that moist air with molar density $N$ is in equilibrium in the vertical dimension as:

$$- \frac{\partial N}{\partial z} = kN, \quad N = N_v + N_d. \tag{3}$$

Here $k$ has the units of inverse height. Hydrostatic equilibrium corresponds to $k = (Mg/R - \partial T/\partial z)/T$. (But note that Equation (3) can be applied to describe physical equilibria of different nature. For example, in a vertically isothermal atmosphere in the absence of gravity $k = 0$.)

Condensation causes the distribution of vapor $N_v$ to deviate from the equilibrium distribution. The condition that moist air as a whole must nevertheless remain in equilibrium causes dry air $N_d$ to also deviate from the equilibrium in the opposite direction to the vapor:

$$- \frac{\partial N_v}{\partial z} = (k + k_v)N_v, \quad - \frac{\partial N_d}{\partial z} = (k + k_d)N_d. \tag{4}$$
\[ k_v N_v + k_d N_d = 0, \quad (5) \]
\[ k_v \equiv -\frac{1}{N_v} \frac{\partial N_v}{\partial z} - k, \quad k_d \equiv -\frac{1}{N_d} \frac{\partial N_d}{\partial z} - k. \quad (6) \]

The value of \( k_v \) describes the intensity of the mass sink. In the case of water vapor \( k_v > 0 \) is caused by a steep vertical temperature gradient that causes vapor to condense\(^1\). (However, the same formalism can be used to describe a mass sink of any nature, e.g., produced by a chemical reaction occurring in the atmosphere.)

Putting (4) into (1) using (5) we obtain:

\[ u \frac{\partial N_d}{\partial x} = -w k_v N_d \left( 1 + \frac{N_v}{N_d} + \frac{S}{w k_v N_v} \right). \quad (7) \]

Given our assumption that \( S \) is linear over \( N_v \) and that in the absence of condensation dry air should be in equilibrium, \( \partial N_d/\partial x \to 0 \) at \( N_v \to 0 \), then Eq. (7) is satisfied only when

\[ S = -w k_v N_v, \quad (8) \]

which is Equation (34) of M10. So in this formulation condensation rate \( S \) is a linear function of three independent variables: vertical velocity \( w \), local amount of vapor \( N_v \) and deviation \( k_v \) of vapor from the equilibrium distribution \( (k_v \) can be characterized as the "condensability strength" of atmospheric vapor). Note an interesting relationship: with \( S \) given by (8) and \( \gamma \equiv N_v/N \) we have \( S_d - S \equiv S \gamma_d \equiv S_d \gamma \).

The physical meaning of the equilibrium condition (3) is that in the presence of condensation one can specify the vertical distribution of \( N \) such that it does not depend on the distribution of the condensable gas (vapor) \( N_v \), \( \partial k/\partial k_v = 0 \). Demanding dry air to be in equilibrium, the same chain of arguments, i.e., assuming that \( S \) is linear over \( N_v \), yields \( S = S_d \) and \( \partial N_d/\partial z = \partial N/\partial z = 0 \). As discussed in Section 4.2 of M10, in such a case the non-equilibrium pressure gradient caused by condensation will be located not in the horizontal, but in the vertical dimension.

Note that in Eq. (1) any small difference of the order of \( \gamma_d \) between \( S \) and \( S_d \) is multiplied by a large magnitude \( 1/\gamma_d \gg 1 \) and thus has a profound influence on the magnitude of the horizontal gradient \( \partial N_d/\partial z \). The lack of theoretical approaches to account for condensation rate \( S \) in the current meteorological literature lead to flawed results. This is well illustrated in modelling studies designed to investigate the precipitation mass sink – as we shall detail in the following comment\(^2\).

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\(^2\)Reply to Dr. Gavin Schmidt on Bryan & Rotunno (2009), \( \text{http://www.atmos-chem-phys-discuss.net/10/24015/2010/acpd-10-24015-2010-discussion.html} \)