Response to Dr. Gao's comments.

The authors would like to thank Dr. Gao for his careful review and many valuable comments and suggestions. We agree with most of the reviewer's comments and changes have been made accordingly to address the reviewer's concerns. A point-by-point response is provided below.

Main comments

1. A few comments on the target wind profiles.

i) The authors fixed the target tangential wind and only varied the radial wind in the two groups of experiments to explore the impact of the radial wind shear. However, in physics-based models one cannot vary the radial wind without changing the tangential wind as the two wind components are intrinsically coupled. Since the authors aimed to explore the turbulence characteristics in an idealized setting, I think their purpose justified their choice. But the authors are recommended to explicitly state this in their experiment design.

We agree. The following statements are included.

"This paper is focused on the radial wind shear because of its direct link to the inflection point instability (GG14). Therefore, only one target tangential wind is prescribed for each simulation group, which has three target radial wind profiles as discussed above. It is recognized that changes in the radial wind inevitably affect the tangential wind. The sensitivity of the LES results to the tangential winds is also explored." (Page 6, line 14-21)

ii) There is no detailed description for how the target wind profiles are obtained. Are these profiles derived from a dynamical model or observed profiles.

These target wind profiles are based on both the wind profiles from the dynamic model of Foster (2005) and observations of Morrison et al. (2005). In the revised manuscript, a new paragraph (see below) has been included to describe how the target wind profiles are obtained.

"The target wind profiles are formulated based on the normalized typical hurricane wind profiles obtained from a dynamical model of Foster (2005) and from the observations by Morrison et al. (2005). The relaxation is used to nudge the LES mean wind toward the target profiles so that the resultant equilibrium mean wind profiles have special characteristics for effective simulation comparisons. We have experimented with dozens of LES simulations using a variety of the wind profiles. The two groups of the target wind profiles (i.e., H and L groups, see Fig. 1) are chosen from these additional trial simulations and they exhibit systematic variations in shear strength and infection point levels to serve our objectives. The target radial wind U_T of H2 and tangential wind V_T of group H generally follow those of Fig. 2 of Foster (2005) except for the HBL height. In addition, the super gradient wind shape is also included in V_T in accordance to Fig 3a of Morrison et al. (2005). The U_T profile of H2 is multiplied by 0.5 and 1.5 to provide U_T for H1 and H3 with the different shear strength but similar IPL, respectively. The target radial wind U_T of L2 is obtained by vertically suppressing U_T of H2 and increasing the near-surface value to 13 m s^{-1}. Then, U_T of L2 is multiplied by 0.5 and 1.5 to give U_T of L1 and L3, respectively. The target tangential wind profile V_T of group L is obtained by lowering the HBL height for V_T of group H." (Page 6, line 1-12)

iii) The authors only showed the LES results under the gradient wind speed of 45.5 m/s. Are the results shown in this manuscript representative for a range of gradient wind speeds that the authors have investigated? Or is the choice of 45.5 m/s somehow arbitrary? Either way, the authors should make it clear.

All the simulations are run using the gradient wind speed of 45.5 m/s, which is somewhat arbitrarily chosen simply because it represents a middle-to-high value for the gradient wind speed in a hurricane environment (e.g. Willoughby 1990).

"Therefore, the same value of 45.5 m s^{-1} is used as it represents a middle-to-high speed range of the gradient wind in a hurricane environment (e.g. Willoughby 1990)." (Page 5, line 29-30)

2. The model produced much higher mixed layer depth (Fig. 2a) than typically observed (Zhang et al., 2011, MWR). This is very likely because some important processes that stabilize the hurricane boundary layer, such as radial advection and diabatic effect (Kepert et al., 2016, JAS), were not considered in this study. The unusual high mixed layer has important implication for the large eddy characteristics. As discussed by Gao and Ginis (2014 and 2016), the height of mixed layer has critical impact on the roll characteristics and their coupling with internal waves. The authors are recommended to add some discussion on this.

We agree with the comments. Indeed, neither radial advection nor diabatic effect is included in the heat balance, which can affect the growth of the mixed layer. This issue is noted in the revised manuscript (see below).

"In addition, z_i has critical impact on the roll characteristics and their coupling with internal waves (GG14), which will be emphasized in discussions of the roll structure in later sections. It also should be noted that the high z_i may also reflect the fact that neither radial advection nor dabatic heating is included in the heat balance. These processes may affect the growth of the mixed layer (Kepert et al., 2016)." (Page 7, line 12-17)

3. The authors did nice quantitative analysis showing that the simulated rolls have a quasi-teo-dimensional structure with the two velocity components (u', w') of the over-turning circulation 90 degree out of phase. While their analysis help understanding why the correlation between u' and w' is poor and the cross-roll momentum flux is weak, there is still lack of a fundamental explanation for the vertical tilting of the convergence zone (Fig. 6). One possible explanation is that as shown in Foster (2005) and Gao and Ginis (2014), the roll streamlines tend to tilt vertical to efficiently extract the kinetic energy from the mean shear flow. The titled convergence zone and the negative cross-roll momentum flux result from the titled roll streamlines. The authors are suggested adding some discussion on this and revising the 4th point in the summary of section 3 accordingly.

We agree with the reviewer that we have not provided a fundamental explanation for the vertical tilting of the convergence zone. GG14 suggested that the tilting results from the tilted roll streamlines, leading to the energy extraction from the radial wind shear. Although the circulation streamlines are different from the convergence zone, the tilting of either perturbation variable implies the same result, that is, the tilting allows the generation of the radial momentum flux. We feel that neither of these titling is more fundamental than the other. The tilted convergence zone is indeed consistent with the tilted stream lines in the sense that it leads to the energy extraction from the mean radial wind shear. In the revised version, this point is emphasized.

"The cross section of the roll circulation from Fig. 6 shows that updrafts are originated along the convergence slope where $u' \cong 0$ and tend to coincide with negative u' above the slope, leading to a downward (or negative) cross-roll momentum transport aloft. The negative momentum flux (i.e. $\overline{w'u'}$) in conjunction with the positive wind shear represents energy production for roll circulations. This result is also consistent with those of Foster (2005) and GG14, which show that the roll streamlines tend to tilt vertically to efficiently extract the kinetic energy from the mean shear flow." (Page 9, line 25-29)

4. The LES results are of central importance for the development of turbulent flux parameterizations under conditions where in-situ observations are difficult to obtain. This work presents important LES results under high wind shear conditions. The authors are encouraged to strengthen their discussion on the turbulence parameterizations under hurricane conditions. There are a few aspects worth attention. The authors are not asked to add a whole new section; one paragraph or two would be sufficient.

We agree with the reviewer that the LES results such as the current study may greatly help developing parameterizations for HBL. Our objective of this work is to study overall roll structure including the mean forcing, statistics, roll pattern, and spectral characteristics. The turbulence parameterization issue is not in our original plan for this work. Nonetheless, we add a subsection to present calculations of some momentum transfer coefficients as explained below.

i) It would be interesting to apply the flux decomposition method to analyze the results from other experiments and compare the large-eddy induced fluxes under the scenarios in which the single-mode roll structure is dominant or not.

It would be indeed a very interesting practice. We, however, have not calculated spectra at each level for each of the simulations, which require processing large amount of dataset. From the spectra at *z/zi*=0.2 (not shown in the paper) and z/z_i=0.4, it is evident that H3 produces the roll scale circulations that are most dominant in all the power spectrum and co-spectra (Fig. 8 and 9). It is expected that the roll fluxes of H3 should be larger than those from other simulations, even though it is not quantitatively proved. Therefore, we decided not to conduct more decomposition analysis for other experiments.

ii) The vertical distribution of the roll-induced radial and tangential momentum fluxes presented in this study seem largely consistent with Gao and Ginis (2016), which investigated the correlation between the roll-induced momentum fluxes and the mean wind shear based on 2-D

model results. It would be of interest to apply the same method and see if the 3-D LES results qualitatively agree with Gao and Ginis (2016).

We agree with the reviewer that the representation of momentum flux is a central issue in the HBL study and LES simulations may provide key understanding on the issue. We present examples of the computed momentum transfer coefficients of three spectral groups based on the spectral analysis of H3 in a new subsection (i.e., Section 4.4). Overall, there are both similarities and differences between those of Gao and Ginis (2016) (GG16, hereafter) and our examples. For instance, although our result shows certain counter-gradient behavior of K_v, similar to that of GG16, this feature is less robust in our case than in GG16. It is because the counter-gradient behavior in our simulation occurs near the HBL top where $\overline{w'v'}$ is significantly smaller than that from GG16, in which the non-local feature occurs in the mid-HBL. This difference is mainly because of the different mean tangential wind profile used in the respective model as discussed in the subsection 4.4.

"4.4 Momentum transfer coefficients

The momentum transfer coefficients, defined by the negative ratio of the momentum flux to the mean wind shear according to the K theory, play a central role in the representation of HBL. It has been shown and/or argued that the roll generated momentum flux cannot be represented by the local transfer theory because of the "large-scale" nature of the roll circulation in terms of its horizontal and vertical scales as compared to z_i (e.g., Foster, 2005; Zhu, 2008; and Gao and Ginis 2016). In this subsection, the issue of the transfer coefficient is briefly discussed using the results from the spectral analysis. Because the momentum fluxes have been decomposed into three spectral groups, it is convenient to compute the transfer coefficient for each group. By definition, the transfer coefficient for the radial momentum flux from each spectral group K_u^i can be calculated by

$$K_u^i = -\frac{\overline{w'u'}^i}{\partial \overline{u}/\partial z}, \tag{3}$$

where superscript $i \in (s, l, r)$ represents the small scale (< 1 km), large-eddy scale (1 – 2.5 km), and roll scale (> 2.5 km). The transfer coefficient for the tangential momentum flux K_v^i is computed in the same fashion. Because both the momentum fluxes (except for $\overline{w'u'}^r$) and the vertical gradient of the wind speed are very close to zero above 1400 m, all the values of the computed transfer coefficients are removed for $z \geq 1400$ m.

These transfer coefficients are shown in Fig. 13. The values of K_u^i change little with height from 200 m to 1.1 km, above which K_u^r increases significantly because of both the finite values of $\overline{w'u'}^r$ and near-zero gradient of $\overline{u}$. The non-zero $\overline{w'u'}^r$ above HBL is likely caused by the internal gravity waves which are connected to the roll structure and have the same wavelength as the rolls as discussed previously (also see GG14). The transfer coefficients K_u^i are ill-defined around z = 200 m because $\partial \overline{u}/\partial z \approx 0$. Unlike the nearly constant K_u^i, the tangential transfer coefficients K_v^i increase with height from zero at surface to ~ 150 m^2 s^{-1} at 850 m. They then sharply increase near the HBL top where $\partial \overline{v}/\partial z \approx 0$, which results in both very large positive and negative values of K_v^i because $\overline{w'v'}^i$ is always negative while $\partial \overline{v}/\partial z$ changes sign. This behavior is contradictory to the downgradient transfer theory which assumes none negative K_v^i (e.g., Stull 1988, pp 108). This is similar to the result of the counter-gradient $\overline{w'v'}$ for the same reason from the two-dimensional roll model of GG16. The main difference is that their counter-gradient feature occurs in the mid-HBL where the momentum flux is significantly larger than that near the HBL top in our simulation of H3 described above (Fig. 10b). This difference is mainly caused by the different mean tangential wind profiles obtained with different methods: the dynamic model approach of GG16 and the mean nudging in this work. Therefore, there is a need to apply the same mean wind profiles in both the 2-D roll and LES models for a more effective comparison. It is also worth noting that the sub-grid scale parameterized flux is not included in either $\overline{w'v'}^i$ or $\overline{w'u'}^i$. The inclusion of the SGS flux would slightly change the small-scale transfer coefficient profiles K_v^s and K_u^s.

Overall, there are marked differences between K_u^i and K_v^i in the mid-HBL between 200 m and 850 m. The values of either K_u^i or K_v^i do not vary greatly between the spectral groups even though the differences are obvious. The counter-gradient feature occurs at the HBL top where $\partial \overline{v}/\partial z$ changes sign and $\overline{w'v'}^i$ remains negative. Its effect on the momentum flux parameterization would be likely negligible, because $\overline{w'v'}$ is very small near the HBL top in the case." (Page 15-16)

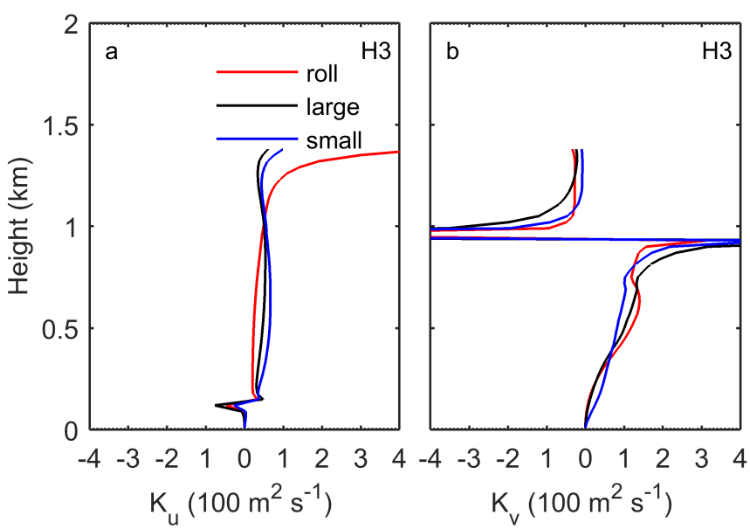

Fig. 13. The momentum transfer coefficients for three spectral groups from H3 for K_u (a) and K_v (b). The three spectral groups are small scale (< 1 km), large-eddy (1-2.5 km), and roll (> 2.5 km), respectively.

Minor comments

Page 2 line 33: Suggest changing "Others neglect the effects by assuming …" to "Others neglect the horizontal advection effect on the HBL wind profiles by assuming …"

The sentence is modified as follows:

"Others neglect the horizontal advection effect by assuming a local balance among the turbulent mixing, gradient wind, Coriolis force, and hurricane induced centripetal force."

Page 3 line 4: Change "Morrison and Bussinger (2005)" to Morrison et al. (2005)

The sentence is changed to

"For example, Morrison et al. (2005) provided both observed radial and tangential winds from WSR-88D radar data …"

Page 4 line 7: The Charnock relationship gives a monotonic increase in the drag coefficient for increasing surface wind speeds, which was found not valid under high wind condition (surface greater than ~ 30 m/s). Did the authors put any constraint on the surface roughness length (drag coefficient) under high wind in this study?

All the simulations use the formulation of Donelan et al. (2004). However, because the simulated 10 m wind speed is usually less than 33 m s^{-1}, which triggers the level-off of the drag coefficient, differences caused by the different drag coefficients (the Donelan vs. the Charnock) should be minimum compared to those by the wind shear. Please see the following added statements in the revised manuscript.

"The surface momentum flux is calculated using the roughness length (z_0) formulation of Donelan et al (2004). That is, z_0 increases with the 10 m wind speed following the Charnock relationship for the wind speed less than 33 m s^{-1}, above which z_0 is set equal to 3.35 mm, which is equivalent to the drag coefficient at 0.0025. Because the 10 m wind is usually less than 33 m s^{-1} for all the simulations, this modification of z_0 on the Charnock relationship should not have major effects on the results presented here." (Page 4, line 6-11)

Page 6 line 5: Foster (2005) used a linearized dynamical model to obtain the wind profiles and did not use observed wind.

The sentence is modified as follows:

"The target wind is formulated based on the typical hurricane wind profiles obtained from a dynamical model by Foster (2005) and the wind observations by Morrison et al. (2005)".

Page 6 line 24: The fact that experiment H3 has the highest mixed layer is likely partially due to the strongest nonlocal mixing effect of rolls/large eddies, which have largest vertical extent in this experiment, not only due to the strongest turbulence intensity.

We agree with the reviewer's comment and add following statement:

"It will be shown in following sections that H3 results in the vigorous roll structure, which is likely to contribute to the highest z_i by strong nonlocal mixing as discussed by GG14." (Page 7, line 12-17)

Page 7, line 30-32 and Page 10, line 1: It is not clear why the authors say H3 has the most vigorous rolls. Is the based on the maximum w' or the domain-integrated kinetic energy? The turbulence statistics shown in Fig. 7 suggest that the maximum w' in L3 (which has strongest radial shear) may be larger than H3.

This is a very good point. The rolls of H3 are called the "strongest" because H3 has the following unique features: (1) the highest z_i (Fig. 2); (2) the strongest turbulence intensity and momentum flux above 500 m (Fig. 7); (3) the largest roll wavelength (Table 1); and (4) the strongest peaks at the roll wavelength for all the turbulence spectra and co-spectra (Fig. 9). Because the spectra are not discussed until section 4, the word "strongest" is removed from the statement on page 7, which now becomes:

"It is noteworthy that H3, which has the highest IPL and moderately strong wind shear (Table 1), is characterized by the vigorous rolls that have the largest horizontal scale, implying the importance of the IPL in regulating the roll intensity as well as the scale." (Page 8, line 19-21).

"The following features associated with H3 are evident: (1) the highest z_i (Fig. 2); (2) the strongest turbulence intensity and momentum fluxes above 500 m (Fig. 7); (3) the largest roll wavelength (Fig. 8 and Table 1); and (4) the strongest peak at the roll wavelength of the turbulence power spectra and co-spectra among all the simulations (Fig. 8 and 9). These features suggest that H3 has produced the most robust roll structure because of the radial wind that has the highest IPL and relatively strong shear (Fig. 2b). " (Page 12, line 21-27)

Page 8, line 17: This sentence needs to be revised since it is somehow counterintuitive by saying "downward transport driven by vigorous upward motion".

The "downward transport" here means that the momentum flux is negative by the common convention of the boundary layer meteorology. Therefore, upward motion ($w' > 0$) can produce downward transport (i.e., $w'u' < 0$) by moving weaker horizontal wind ($u' < 0$) upward. Similarly, downward motion ($w' < 0$) can produce downward transport (i.e., $w'u' < 0$) by moving stronger horizontal wind ($u' > 0$) downward. For clarification, the sentence is revised to

"These patterns suggest that the roll-scale tangential momentum flux is dominated by the downward transport (i.e. negative momentum flux) driven by the vigorous upward motion." (Page 9, line 4-5)

Page 15 line 10: It is not clear how the tangential momentum flux $w'v'$ affect the turbulence intensity.

First, the momentum flux $\overline{w'v'}$ directly contributes to the total turbulence kinetic energy, $e = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$, through the shear term ($-\overline{w'v'} \cdot \partial\overline{v}/\partial z$). Second, there is strong energy transfer between $\overline{u'^2}$, $\overline{v'^2}$, and $\overline{w'^2}$ through the pressure-velocity covariance terms in their respective variance budget equation, i.e., $p'\partial v'/\partial y$ in $\partial\overline{v'^2}/\partial t$, $p'\partial u'/\partial x$ in $\partial\overline{u'^2}/\partial t$, and $p'\partial w'/\partial z$ in $\partial\overline{w'^2}/\partial t$. Therefore $\overline{w'v'}$ may indirectly affect both $\overline{u'^2}$ and $\overline{w'^2}$, which represent the roll circulation intensity. The following is the modified text.

"The above results suggest that the radial wind shear plays a more dominant role in determining the roll characteristics with regard to the scale selection, while the tangential wind shear strongly influences the tangential momentum flux $\overline{w'v'}$. Consequently, the tangential wind shear enhances both the overall turbulence intensity, $e = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$, through the shear production. It also can affect the roll circulation, $(\overline{u'^2} + \overline{w'^2})$, through the return-to-isotropy terms in the variance equations as shown in NN12." (Page 17, line 13-15

Page 17 line 23: While it is true that rotation terms have no significant direct impact on rolls generated by shear instability, Foster (2005) suggested that at small radii (inside of the radius of maximum wind) another type of instability associated with the rotation terms (the parallel instability) may be dominant mechanism for roll formation.

We agree with the reviewer and modify the statement accordingly.

"These results confirm the previous two-dimensional model simulations and LES analyses that the rotation terms do not have major influence on the turbulence structure, which is largely driven by the wind shear, although these terms may play the dominant role in the case of the parallel instability (Foster 2005)." (page 20, line 3-5).

Figure 3 and 4: At what time are these snapshots selected?

These snapshots are chosen at 9 hour. "9 hour" is included in the figure caption.

"Figure 3: Horizontal cross sections of vertical velocity $\mathbf{w}'$ at 9 hour at three different levels (i.e., z/z_i = 0.2, 0.4, and 0.9, respectively) from group L and H simulations."

"Figure 4: Plan views of turbulent perturbations at 9 hour from H3 at z/z_i=0.2. The fields are (a) v', (b) $w'v'$, (c) u', and (d) $w'u'$. An "eye-fit" black line is drawn in (c) to show an example of convergence zone induced by the radial wind."

Figure 4: There is no black line in (c). Also, the caption for (d) should be $w'u'$.

The black line is drawn on Fig. 4c and $w'v'$ is changed to $w'u'$.